Communication and Academic Vocabulary in Mathematics: A Content Analysis of Prompts Eliciting Written Responses in Two Elementary Mathematics Textbooks

by

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Abstract

The purpose of this study was to investigate how writing in mathematics is treated in one $4th$ grade National Science Foundation (NSF)-funded mathematics textbook titled

learning of mathematical concepts through writing, potentially impacting student performance on national and international assessments.

Chapter 1: Introduction

A Vignette

As an elementary-

observations (Downey, English, Frase, Poston, & Steffy, 2004) of classrooms to gather evidence of best practices in mathematics instruction. In doing so, I collaborated with the literacy coach and noticed a discrepancy between the walk-through checklists for mathematics versus literacy. According to the county-produced literacy checklist,

and high frequency), conferring notes for writing, conferring notes for reading, leveled classroom libraries, book baggies with accountability forms, student writing samples on the bulletin board, leveled reading groups, and anchor charts. Conversely, the math checklist asked evaluators to find evidence of the district-adopted calendar kits and readily available manipulatives. Unlike the literacy checklist, the mathematics checklist did not include evidence of teacher use of these materials or any other instructional practice for mathematics. Where was the math word wall with content strand vocabulary? Where were the student math writings on bulletin boards (e.g., math stories, strategies for solving a problem, solution steps, explanations, and justifications)? Where was the math

support the mathematics topic? Where was the evidence of student conferencing notes regarding how students solved problems (i.e., documentation of strengths and

weaknesses)? Where was evidence of the math groups? Where were the anchor charts for alternative and traditional strategy solutions? Where was the math?

As a math coach, my support for teachers centered on the content standards and small group instruction. This support was guided by the most pervasive resource in the mathematics classroom-- the textbook. My conversations with teachers primarily focused on how I could assist teachers in designing purposeful activities for small group instruction. From those conversations I developed activities for multiple grade levels throughout my school. Most of the activities centered on integrating mathematics writing through problem solving, journaling, and real world application of mathematics (i.e., newspapers). I also used technology, making sure each student had a spiral notebook to solve problems and write down the solution steps to the problems they answered on the computer. Interestingly, every activity I developed for small group instruction, for multiple grade levels, incorporated writing. After reflecting on my experiences of the

to understand that my coaching philosophy for teachers was centered on the process standard of communication, more specifically, that of writing.

ered on writing, I began

A Case for Writing in Mathematics

The use of writing in mathematics teaching aligns with the recommendations of the National Council for Teachers of Mathematics (NCTM) process standards. The NCTM *Principles and Standards for School Mathematics* (PSSM) states that mathematics content standards are learned through five process standards: problem solving, reasoning and proof, communication, connections, and representation. Although the process of communication appeared to address my implementation of writing in

Various organizations, such as the National Council of Teachers of Mathematics (NCTM), National Research Council (NRC), and members of the Council of Chief State School Officers (CCSSO) and the National Governor s Association, Center for Best Practices (NGA Center), have produced standards documents that highlight the use of writing in the mathematics classroom. For example the NCTM identified five process standards in the *Principles and Standards for School Mathematics* (NCTM, 2000). The NRC formulated the *Strands of Mathematical Proficiency* (NRC, 2001). Furthermore, members of the Council of Chief State School Officers (CCSSO) and the National Governors Association, Center for Best Practices (NGA Center) developed common standards for all states where communication is embedded throughout the content recommendation (CCSS, 2010).

The NCTM *Principles and Standards for School Mathematics* (2000) states that the content strands (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) should be taught through mathematical processes (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation). Whether the processes are utilized in isolation or as a connected component, the process of writing can be demonstrated throughout these strands. For example, in order to problem solve one can write an explanation or description of the problem solving process mathematical thinking. Students can also write to

describe the process of connecting the mathematics content in addition to providing an explanation of a particular mathematical representation.

The textbook publishing industry, as well as curriculum projects funded by the National Science Foundation (NSF), moved quickly to develop curriculum materials (i.e.,

textbooks) to align to standards recommendations from these various organizations. Publishers realize that in addition to the standards documents, the most common influence on content appears to be the textbook/curriculum program (Weis, Pasley, Smith, Banilower, & Heck, 2003). Thus, the mathematics textbook is typically researched as the dominant tool in classroom instruction (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr et al., 2008).

Statement of the Problem

Although curriculum development projects, oftempe/Pa13gae

writing prompts in elementary mathematics textbooks is warranted. The following task is an example of a writing prompt used for analysis:

How do you know 1/4 is greater than 1/5? Explain your thinking.

(Urquhart, 2009)

I selected two elementary $4th$ grade textbooks with teacher editions: (1) the 2011 edition of *enVision MATH* published by Pearson Education, Inc. and (2) the third edition of books developed by the University of Chicago School Mathematics Project (UCSMP), funded by the National Science Foundation (NSF) titled *Everyday Mathematics, Common Core Edition*. Both of these textbooks were national versions and were therefore not modified to fit the needs of any one specific state.

Purpose of the Study

The purpose of this study is to examine writing prompts in mathematics textbooks. Specifically, I will explore the following questions:

- 1. How many writing prompts are included in one $4th$ grade NSF-funded mathematics textbook and one publisher-generated mathematics textbook?
- 2. How do mathematical writing prompts vary across the content strands between one 4th grade NSF-funded textbook and one publisher-generated textbook?
- 3. What types of vocabulary are used in the writing prompts in one $4th$ grade NSFfunded mathematics textbook and one publisher-generated textbook?
- 4. What types of prompts are provided in one $4th$ grade NSF-funded mathematics textbook and one publisher-generated textbook?

Theoretical framework. I conducted this study through the lenses of three interwoven theoretical perspectives: cognitive, social,

. In support of this perspective, many organizations [e.g., NCTM,

NRC, Writing to Learn (WTL) activities -

following areas: the structure of language and the audience or purpose for the writing task. For example,

content strands, (2) Baumann and Graves (2010) classification scheme of academic vocabulary, and (3) research in mathematics writing prompt types (Burns, 2004; Dougherty, 1996; Urquhart, 2009; Whitin & Whitin, 2000) (see Appendix A). Using the framework as a way to record the data, I calculated the number of writing prompts per page, the number of exercises per page, page number, and the wording of the prompt. Then I further coded the prompt to determine the academic vocabulary used, and the total5so8 TJETBT

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general vocabulary, meta-language, symbols, prompt/writing task, and *constructed response.*

Academic vocabulary. Baumann and s (2010) note that academic vocabulary is defined in two ways: 1) *domain specific* or the content used in disciplines like mathematics, and 2) *general academic* or the broad, all-purpose terms that appear across content areas but that may have different meanings depending on the context. In a

of literature use to describe characters, settings, and characters problems and actions*, meta-language* or the terms used to describe the language of literacy and literacy

instruction and words used to describe processes*,* and *symbols* or icons that are not conventional words*.*

Constructed response*.* A *Constructed Response* is an *open-ended* item in which students create or produce an answer or response in written form (McMillan, 2004). These types of items are different from *close-ended* items whereby the answer is selected other technical materials (p. 9). Marzano and Pickering (2005) devised a *Building* word list whereby 7,923 terms in 11 subject

areas were extracted from national standards documents. These lists contain content specific words that are organized into four grade-level intervals where 86 of the terms are specific to the domain of mathematics. For purposes of this study, domain specific academic vocabulary has been modified to *domain specific vocabulary (DSV).*

General vocabulary*.* Baumann and Graves (2010) define General Academic Vocabulary as words that appear reasonably frequently within and across academic domains. The words may be polysemous with different definitions being relevant to different domains (p. 9). In addition, Coxhead (2000) developed an Academic word list based on terms that are most often found in academic texts. For purposes of this study, general academic vocabulary has been modified to *general vocabulary (GV).*

Meta-language*.* Based on the extant work on typologies of academic vocabulary, Baumann and Graves (2010) defined *meta-language* as terms used to describe the language of literacy and literacy instruction and words used to describe processes, structures, or concepts commonly included in content area texts (p.10). Marzano and Pickering (2005) **Building Academic Vocabulary Teacher's Manual Academic Vocabulary Teacher's Manual Word list was also** used for terms that are specific to *meta-language.* These word lists detail content specific vocabulary organized into four grade-level intervals. These terms are specific to describing processes in mathematics writing prompts in the written (textbook) curriculum that have the potential to facilitate writing.

Prompts/writing task. The term *prompt* is used interchangeably with *writing task* in this study*.* Research in the field of literacy and mathematics also uses the terms

prompt and *writing task*

Chapter 2: Literature Review

Integrating literacy practices into mathematics is recommended by reform efforts

teaching mathematical concepts. More specifically, the

NCTM (2000) recommends using the process strand of communication (both written and

(NCTM, 1989, p. 2). One theme common to the NCTM Standards and to the recent changes in

29).

Principles and standards for school mathematics. Another document that impacted the development of curriculum materials was the production of the *Principles and Standards for School Mathematics* (NCTM, 2000). This document updated the 1989 *Curriculum and Evaluation Standards* while building an emphasis on teaching the content strands (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability)

Select and use various types of reasoning and methods of proof (NCTM, 2000, p. 56).

Communication:

Organize and consolidate their mathematical thinking through communication Communicate their mathematical thinking coherently and clearly to peers, teachers, and others

Analyze and evaluate the mathematical thinking and strategies of others

Use the language of mathematics to express mathematical ideas precisely

(NCTM, 2000, p. 60).

Connections:

Recognize and use connections among mathematical ideas

Understand how mathematical ideas interconnect and build on one another to

produce a coherent whole

Recognize and apply mathematics in contexts outside of mathematics (NCTM,

2000, p. 64).

Representation:

Create and use representations to organize, record, and communicate mathematical ideas.

Select, apply, and translate among mathematical representations to solve problems.

Use representations to model and interpret physical, social, and mathem

ade-level curriculum focal points and

connections is to enable students to learn the content in the context of a focused and

(p. 10). The *Curriculum Focal Points* are similar to the *Principles and Standards for School Mathematics*
Productive disposition, the belief that mathematics makes sense and is useful (NRC, 2001, p. 116).

in mathematics, the

support for writing is evident.

Common Core Standards*.* The release of the Common Core State Standards (CCSS) is an effort to promote democracy, equity, and economic competitiveness in the standards movement that began over 20 years ago during the publication of the NCTM *Curriculum and Evaluation Standards for School Mathematics*. In 2010 the NCTM, the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE) produced a joint public statement regarding the support of the implementation of CCSS by stating:

By initiating the development of the CCSS, state leaders acknowledged that common K grade 8 and high school standards culminating in college and career readiness would offer better support for national improvement in mathematics achievement than our current system of individual state standards. The CCSS provides the foundation for the development of more focused and coherent

mathematical concepts and acquisition of fundamental reasoning habits, in addition to their fluency with skills. Most important, the CCSS will enable teachers and education leaders to focus on improving teaching and learning, which is critical to ensuring that all students have access to a high-quality mathematics program and the support that they need to be successful (National

Council of Teachers of Mathematics, Common Core Standards Joint Statement, 2010, para. 2).

In 2009, 48 states adopted the CCSS and established goals of implementing standards to include directives of the initiative

ion, para.1). The CCSS developed a set of standards titled, *Standards for Mathematical Practice* integrating the components of the

Use appropriate tools strategically; mathematically proficient students consider the available tools when solving a mathematical problem.

Attend to precision; mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose. Look for and make use of structure; mathematically proficient students look closely to discern a pattern or structure.

Look for and express regularity in repeated reasoning; mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts (Common Core State Standards Initiative, 2010, p.6).

oficiency

standards, the CCSS recommendations have the process of writing nested within each of the Standards for Mathematical Practice while specifically stating the importance of the acquisition of symbols for proficiency. Clearly the NCTM, NRC and CCSS recommendations have the potential to utilize the process of writing within the learning of mathematics.

In the area of curriculum, the Standards recommendations provide the framework for curriculum and instructional development. In support of standards and reform in curriculum materials, Pattison and Berkas (2000) note that the process of integrating standards into the curriculum emphasizes learning and growth for all as the natural and desired outcome of reform in the schools.

Summary. Reform recommendations for school mathematics resulted in the development of standards documents from the National Council for Teachers of

Mathematics, the National Research Council, and the members of the Council of Chief State School Officers and the National Governors Association. In analyzing these standards documents, a common thread among these resources is that in order for students to become mathematically proficient students must be able to *reason* mathematically. Consequently, mathematics instruction should focus on strategies that utilize the process of reasoning. If instruction focuses on the process of *reasoning* specifically, the mathematical standards from the various sources will be adhered to effortlessly. Although there is some reference to writing mathematically in the standards, using writing in the service of learning mathematics can be utilized as a strategic method for mathematical proficiency in most every standard developed.

Mathematics Textbooks

The mathematics textbook is an important tool in the mathematics classroom. The mathematics textbook is developed based on the standards and recommendations from various documents and reports regarding research in mathematics teaching and learning. Because the textbook is the dominant tool in the mathematics classroom (Hagarty $\&$ Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr, et al., 2008) with direct claims to an alignment with standards recommendations, an analysis their of openended, writing prompts is warranted.

In an effort to investigate the types of prompts in a mathematics textbook, it is important to understand two components of mathematics curriculum: (1) forces that impact major developments in the mathematics textbook; and (2) research in the area of mathematics textbook content analysis. A review of these two components is included in the following section.

Development of textbooks aligned to standards. In the mid to late 19

feedback to the writers, and the commercial publishers who produced and distributed the completed curricula (Reys, Robinson, Sconiers, & Mark, 1999).

Development of the mathematics textbook. One of the major influences on content and instruction is textbook/curriculum programs (Weiss et al., 2003). As states adopted the standards that reflected the NCTM vision, the publishing industry moved quickly to make adaptations to their textbooks (Stein, Remillard, & Smith, 2007). More recently, the publishing industry has revised their textbooks to include the CCSS. For example Pearson Scott Foresman (2011) notes:

Only Pearson offers complete and cohesive support to implement the new Common Core Standards and provide the easiest possible transition. We combine

evolving and continually improving instructional materials, content experts and professional development to help you, your teachers, and your students succeed at every step along the way (Pearson, 2011, n.p).

In addition, *Everyday Mathematics* (2010), a National Science Foundation funded curriculum project textbook notes alignment to the CCSS by stating:

We believe these new standards present us with a wonderful opportunity to continue to refine and improve *Everyday Mathematics*, as we have done over many years and three editions. By summer 2011, McGraw-Hill Education will publish the *Everyday Mathematics Common Core State Standards Edition* (©2012). This updated edition will include new and revised lessons at every grade level to ensure that *Everyday Mathematics* meets and exceeds CCSS. The *Everyday Mathematics CCSS Edition* will provide a comprehensive set of print

(*Everyday Mathematics*, 2010, n.p.).

Although textbook companies are adhering to the recommendations currently, this was not always a focus. Traditionally, m

and their relationship to student learning were not viewed as important aspects of scholarly investigation (Grouws, 1992). However, two factors assisted in changing this view. The first factor relates to the research in the area of instructional support regarding the role of the textbooks as a dominant tool in mathematics instruction (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr, et al., 2008). Secondly, national reports regarding student achievement garnered attention for the role and use of the textbook in the classroom.

Textbooks and teachers' use*.* The textbook is used in many facets in the mathematics classroom. The mathematics textbook is not only researched as the dominant tool used in mathematics instruction, but also has the value of providing professional development in mathematics content. The 2000 National Survey of Science and Mathematics Education investigated the use of the textbook in K-12 classrooms. The findings from the survey data indicated that commercially published materials were used in 87% of classrooms in grades K-4 and 97% of classrooms grades 5-8 (p. 81). According to the survey data, Weiss, Pasley, Smith, Banilower and Heck (2003) found that *Everyday Mathematics* published by McGraw-Hill/Merrill Company, and *enVision MATH* published by Addison Wesley Longman, Inc/Scott Foresman, had significant market share (over 50%) in both elementary and middle mathematics school curriculum. Additionally, they reported that 71% of lessons in the textbook were used for

the fundamental findings in this study was acknowledging the use of language or mathematics vocabulary in each of the materials. Although this study was conducted for middle grades textbooks on the content strand of geometry, the study indicates the importance of acknowledging mathematics language and vocabulary in the textbook.

In another exploration of the vocabulary of mathematics prompts, Herbel-Eisenmann (2007) investigated the voice in the mathematics textbook by identifying and categorizing words in one NSF funded student edition, *Thinking with Mathematical Models (TMM)*. By investigating the linguistic choices made by the textbook authors, the researcher categorized words based on four categories: imperatives, pronouns, modal verbs and expressions. HerbelEisenmann's investigation (2007) herbedinvestigation (2007) herbedthe importance of language choice to achieving some of the goals of the Standards. This study also provided a window into investigating how the process standards were situated in mathematics textbooks. However, the focus of the study was on understanding the language to determine the voice of the mathematics textbook, not necessarily a focus on

student learning or teacher development.

Summary*.* Research on textbooks has consisted primarily of middle and high school textbooks consisting of a review of content strands. In agreement, Johnson (2010) noted that studies of mathematics textbooks generally focus on a single content area, such as data analysis, probability, or reasoning and proof. The limited research in this area of process standard investigation needs to be addressed. In addition the paucity of research on content analyses of elementary grades textbooks is limited. An emphasis on the role of the textbook and research investigating vocabulary in the prompts of mathematics textbooks is warranted.

Baxter, Woodward and Olson (2001) note that *math journals* are intended to reinforce mathematics concepts by describing or explaining mathematical ideas or reasoning. In *journal writing,* the student would write about opinions or feelings regarding the mathematics content (Shield & Galbraith, 1998).

Prompts for journals. In journal writing, the prompts consist of a task in which students write about opinions and feelings, that is, an *affective* prompt (Baxter et al., 2001; Shield & Galbraith, 1998). Another type of journal writing prompt is a *narrative* prompt. However, in math journals the writing prompt consists of a task that has expository purposes such as describing or explaining a mathematical process or content. Aspinwall and Aspinwall (2003) conducted a study with 23 fifth-grade students regarding

provide information regarding feelings or attitudes of particular mathematics topics. An analysis of the prompt responses provided findings that these three particular types of mathematics prompts provided the students with a resource to assess their growth, and instructional benefits of detecting trends from within and across the mathematics classes regarding the progression of comprehension of particular topics, skills, concepts, and attitudes/beliefs using beginning of the year and end of the year assessments.

Collaborative journals. In a self-study, Fequa (1997) explored math journals with her kindergarten class. The teacher became interested in how to enhance her understanding of math concepts. While reflecting on her own classroom practice and student learning, the teacher decided to use a large book (*big book journal)* for a class math journal rather than using individual journals. Using a big book journal alleviated First, the activity differed from the traditional individual

writing assignment, and second, it focused on real problem solving in their classroom, The findings from using the big

book journal were many. Students interacted as they discussed how to solve a problem and the teacher recorded the student responses. The journal also provided students with the opportunity to think about and use various symbols (including letters, words and mathematical drawings). The journal also allowed students to represent their thoughts in a meaningful way while being actively involved in reasoning, comparing and counting.

Powell (1997) also found journals to be a useful tool in the mathematics classroom. This classroom study actually analyzed responses in journals that related to the Greatest Common Factor (GCF) and the Least Common Multiple (LCM). The method to collect the data was done qualitatively by reviewing the responses noted in the

journals of the students. The findings suggest that journaling captured the verbal representation of student thinking. Journaling provided the teacher a way to capture,

ing. In this study journaling also

provided an opportunity for students to reflect on mathematical experiences, to examine their written reflections, and to reflect on their ideas critically. This type of reflective thinking enabled the student to become an active learner. Through the use of journaling in this case study, the researcher noted that the writing helped the students develop confidence in their understanding of mathematics and become more thoroughly engaged with mathematics.

Short response. Scheibelhut (1994) conducted a classroom project with first f writing in mathematics.

Students were asked to solve various problems and respond to various affective questions regarding mathematics in short response formats. After reviewing the responses of the first-grade students writing, the preservice teacher was convinced that incorporating writing into mathematics had many advantages. Through writing, the children were able to make sense out of mathematics and recognize its relationship to their everyday lives. The writing of the students also provided the pre-service teachers with insight into the attitudes and needs of the individual students and may have uncovered reasons for mathematics anxiety.

Writing and problem solving (k-12). Using writing to solve a mathematical problem can range from listing steps in the solution process to justifying why an answer is correct. Cognitively Guided Instruction (CGI) is a developmental program based on

reasoning, understanding the reasoning, and teaching in a manner that reflects this knowledge, teachers can and will provide children with a mathematics education better than if they did not have this knowledge (Sowder, 2007). Therefore, student reasoning in verbal or written form provide

exists and serves as a guide for future instruction.

For example, Parker (2007) used the philosophy of CGI with a mathematics curriculum to assist 32 second-grade students to improve their ability to justify solutions to word problems in writing. Over a four week period, students were given mathematics story problems to solve where the explanation process of the solution was the focus. The onses was investigated. The

method of analysis used to score the responses on the pre-and post tests was taken from the framework developed in the Wisconsin Knowledge and Concepts Examination (WKCE) criterion referenced test scoring rubric. The findings suggest that oral sharing of strategies aided the transition to written expression. In addition, the students with both low and high reading ability developed language for expressing thoughts mathematically.

Evans (1984) examined the use of writing to problem solve in short response format. The researchers were two fellow fifth grade teachers. One classroom was an experimental group while the other was a control group. CTBS scores were analyzed from both classes. The scores showed that the control group achieved higher scores due to a gifted population of about six students. The experimental group used writing with computation during math instruction. The control group used no writing during math instruction. Writing in the experimental group consisted of two methods: how to perform a computation and definitions. The findings suggest that the students with the lowest

pretest scores in the experimental group made the most gains. It was further noted from the findings that writing gave the researchers one more tool to help less capable students grow.

Writing and oral discourse. Steele

aspects of the problem solving process. These writing responses also provided the teacher with examples of mathematical thinking to share with other students and also provided the teacher with information to make instructional decisions about the abilities of the students.

In a second study,

their mathematical problem solving processes showed evidence of a metacognitive framework. Twenty ninth-grade algebra students provided written descriptions of their problem-solving processes as they worked with six selected mathematics problems. Qualitative responses were classified in groups and subgroups based on similarity, orientation, organization, execution and verification. The findings suggest that a metacognitive framework was present in the writing of the subjects. Additionally, the findings supported

for teachers to assess how their students learn and think about mathematics.

Steele (2005) study explored the use of writing to help students develop schemata for algebraic thinking within one month. Schema knowledge consists of identification, elaboration, planning and execution of knowledge. Eight seventh-grade pre-algebra students participated in a teaching experiment in which they solved algebraic problems related in mathematical structure. The students were given problems to solve individually, then to write about their thinking by reflecting. Students then met in small groups to discuss their problem solving approaches. Qualitative methods of data analysis were implemented to determine the effectiveness of writing to develop schema knowledge. Interviews and field notes were organized based on patterns and themes. The findings suggest that through explaining in writing the generalizable patterns in

relationships between the quantities in the problems, they made their algebraic thinking explicit. This explicitness helped the students to develop schemata knowledge needed for solving similar algebraic problems.

Writing and assessment. Bolte (1997) examined the combined use of concept maps and interpretive essays as a method of assessment in three mathematics courses. The population studied consisted of 23 prospective elementary teachers enrolled in a mathematics content course, 63 students enrolled in a Calculus I course, and 17 prospective secondary mathematics teachers enrolled in a Survey of Geometries course. The students were asked to construct a concept map regarding a list of terms related to a familiar topic. After the concept map was completed, the students wrote an accompanying interpretive essay in which they clarified and developed the relationships expressed on the map. The essays were to give students the opportunity to reflect on the relationships illustrated on their concept map and refine their thoughts. Each concept map and interpretive essay was scored using an holistic scoring criteria. The concept map

on organization. The findings suggested that the combined use of these instruments provided substantial insight into the degree of connectedness of

periodically in mathematics for grades 4, 8, and 12 (IES, 2010). The framework used for the NAEP assessments consists of five content areas (number/operations, measurement, geometry, data analysis/statistics/probability, and algebra). The questions are submitted in two formats: multiple choice and constructed response.

knowledge is *interrelated* in that knowledge of one word (e.g., *urban*) connects to knowledge of other words (e.g., *suburban*, *urbanite*, *urbane*).

The following sections will explain the different types of mathematics vocabulary and the various mathematical signs/symbols important for mathematical writing prompts in the written (textbook) curriculum that facilitate a constructed response.

Domain specific vocabulary. According to Baumann & Graves (2010), academic vocabulary was found in content area textbooks and other technical writing and can be classified in two ways. The first definition is recognized as *domain specific academic vocabulary*, i.e., content specific words used in different domains such as geometry, biology, civics and geography. Brozo and Simpson (2007) define academic vocabulary as word knowledge that makes it possible for students to engage with, produce, and talk about texts that are valued in school. These words have been referred to as technical vocabulary (Fisher & Frey, 2008) or content specific vocabulary (Hiebert & Lubliner, 2008) or as Tier 3 words (Beck, Mckeowen, and Kucan, 2002). Graves and Bauman (2010) provide the following terms as examples of *domain specific vocabulary* according to their classification scheme*: apex, bisect, geometry, polyhedron, Pythagorean Theorem,* scalene triangle.¹ For purposes of this study, Domain Specific Academic Vocabulary has been modified to *domain specific vocabulary (DSV).*

challenging to learn and use during communication efforts because, depending on the domain, the word will have different meanings. For example, Hiebert and Lubliner (2008)

synthesize. The terms *describe, narrate, reflect/question, and synthesize* were not listed in the Coxhead (2000) word list. However, further analysis of the words placed these terms in the category of *meta-language* in Graves and Bauman's estication scheme. These words all describe processes in mathematics *and* have the *same* meaning across different domains hence the definition of meta-language.

Using mathematical language to communicate is a complex process. In order to achieve this task, students need to be familiar with not only mathematics vocabulary including meta-language, but also signs and symbols. In understanding the nature of signs and symbols in mathematics communication, the field of semiotics is discussed.

Signs and symbols. Understanding how semiotics relates to the field of mathematics communication is important for instructional purposes. Historically, the definition of semiotics began with the philosopher Charles Sanders Peirce who discussed

the sign or signifier (conveys information); a signified (an object or idea that the sign is related throughout), and lastly, an interpretant (which is an interpreted further sign of the object) defining a three part system of meaning (Malcolm and Goguen, 1998). Discourse occurs when the sign receiver (listener or reader) understands the information that the sign producer (speaker or writer) intends to convey (Thompson, et.al, 2008). Similarly, Pirie (1998) lists symbolic language (using mathematics symbols) as one of the means to communicate in mathematics. In addition to acquiring meaning of vocabulary in a written mathematics prompt, the mathematics learner also has to acquire meaning of mathematical signs and symbols in order to achieve mathematical literacy. The complexity of learning and communicating math symbols and words is similar and

should be treated with the same understanding as learning a foreign language. For

example, Thompson et al. (2008) classify a math student as a mathematic 18 Tme37 681.58 Tmg5(rst0 1

word classification system will be

maintained, "We teach a subject not to teach little living libraries on the subject, but rather to get a student to think mathematically for himself (sic)... to take part in the process of knowledge-getting. Knowledge is a process not a product" (p. 72).

From a socio-cultural perspective, mathematics tasks that facilitate written responses also have the potential to facilitate discourse in oral form. Baxter et al. (2001) suggest that written assignments that encourage students to justify and explain problem solutions have the potential to support and extend oral conversations. In support of this notion, Bereiter and Scardamalia (1987) note that empirical support from studies has shown that children write longer texts and texts of higher quality when they are provided ute, 1986; Daiute & Dalton, 1993).

oral discourse and schema building:

As students struggle to get their thoughts into words, they are challenged to process the ideas in order to restate them, elaborate on them, or conjecture about

order to modify and refine their knowledge (p. 1).

Supporting the importance for writing in mathematics, Connolly & Vilardi (1989) claim that writing develops thought processes useful in doing mathematics: abilities to define, classify, or summarize; methods of close, reactive reading; meta-cognition (an

identification of mistakes and errors. Regarding the different ways writing can be used in the mathematics classroom, cognitive, social as well as rhetorical perspectives in terms of

audience and purpose are nested within the constructed response. Cognitive, social and rhetorical theories of writing also define theoretical implications of writing in mathematics.

Writing To Learn

Writing is an important component across academic disciplines in education. The influence of writing as an instructional tool in the mathematics curriculum was highlighted

movement. Romberger (2000) defines WAC as a pedagogical movement that began as a response to a perceived deficiency in literacy among college students. WAC is premised on theories that maintain that writing is a valuable learning tool that can help students synthesize, analyze, and apply course content. Within this movement, writing to communicate

2004). Elbow and Sorcinelli (2006) acknowledge some of the cognitive factors by stating how low stakes writing (a type of freewriting that is used more informally and tends to be ungraded) has the potential to facilitate student reflection, their discovery of new knowledge, their ability to draw connections, and develop metacognitive skills and uncover new ideas without having the fear of being graded.

Forsman (1985) provided a practical rationale for wri

was used as a tool for evaluation. Using this method, teachers used student writing as a means to assess what students have learned.

Similarly, Nuckles, Hubner, Dumer, and Renkl (2010) discuss the findings regarding two longitudinal studies that investigated journal writing while reporting an expertise reversal effect. In the experimental groups, students wrote regular journal entries over a term while receiving a combination of cognitive and metacognitive prompts. Initially, the control group received no prompts. The findings from the data (analyzed using a SOLO taxonomy ranging from six levels of knowledge), suggest that the experimental group applied more cognitive and metacognitive strategies in their journals and showed higher learning outcomes than the control group. The experimental group also showed increasingly higher performance ratings on the mid-year assessment than the control group. However, towards the end of the semester, the writers in the experimental group scored lower than the control group. The researchers describe this negative impact as the expertise reversal effect. In the study, this type of effect describes how the external guidance of prompts was beneficial initially during instruction, but later The implications from this type of

effect can have a negative impact in cognitive and motivational factors in learning. The researchers believe that more research is needed regarding the extraneous factors of

Through the National Writing Project, Nagin (2003) notes that writing is a tool for thinking while emphasizing how the facilitation of such instruction can foster active learning and critical reflection. More specifically,

than just a skill or talent, it is a means of inquiry and expression for learning in all grades

metacognitive stance by supporting the monitoring of comprehension and evaluation of learning outcomes (Nuckles et al., 2010).

Summary

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A review of the research regarding mathematical standards developed in support of reform recommendations underscores the importance of utilizing mathematical process standards to acquire mathematics content. More specifically, the process of *reasoning* was found as a central component in attaining mathematics proficiency throughout the various standards documents. Through the process of writing to reason mathematically, it appeared the additional process standards would be adhered to logically. Furthermore, the standards documents also provide textbook publishing companies with a type of framework for the development of the content within the mathematics textbook. Because mathematics textbooks were found to be a

how the prompts were compiled and/or what resources were used for the prompts) and the mathematical language necessary for communication were not discussed in the findings of the literature reviewed.

In light of these findings, the research questions developed for this study were addressed using an analytic framework developed from the research literature (see Appendix A).
- 2. How do mathematical writing prompts vary across the content strands between one 4th grade NSF-funded textbook and one publisher-generated textbook?
- 3. What types of vocabulary are used in the writing prompts in one $4th$ grade NSFfunded

mathematics textbook and one publisher-generated textbook?

4. What types of prompts are provided in one $4th$ grade NSF-funded mathematics textbook and one publisher-generated textbook?

This chapter consists of seven sections. The first section describes the methods used for textbook sample selection. The second section explains the selection of writing prompts used for analysis. The third section illustrates how the analytic framework was developed through the use of a pilot study. The fourth section describes each of the framework dimensions. The fifth section reveals the parts of the textbooks used for analysis. The sixth section explains the check-coding system used for determining reliability of the framework dimensions. The final section discusses the sources of influence for determining reliability.

Textbook Sample Selection

The selection of textbooks occurred in two phases. In the first phase, I considered the grade level of the textbook to analyze. In the second phase, I considered the specific textbook.

Grade level selection*.* In selecting mathematics textbooks for the study, I considered the results of my literature review and my experience as a mathematics coach. The majority of published textbook analyses were conducted in middle and upper grade

standards documents is a critical question that many states investigate when adopting textbooks (Reyes & Reyes, 2006). More specifically, the professed future alignment of the textbooks to the Common Core State Standards (CCSS) contributed to my selection of textbooks as well. Brief descriptions of these three criteria are explained below.

Widely-used textbooks with significant market share. Textbooks that are classified as widely-used have significant market share if a large percentage of states in the nation adopt the textbook series produced by the publisher (Jones, 2004; Tarr et al., 2008). According to the 2000 National Survey of Science and Mathematics Education (funded by NSF and conducted by Horizon Research Inc.) *Everyday Mathematics* published by McGraw-Hill/Merrill Company and *enVision MATH* published by Addison Wesley Longman, Inc. /Scott Foresman accounted for over 50% of the textbook usage in grades K-4 mathematics classes nationally (Weiss et al., 2003). Therefore, these to findings of the survey data.

NSF and non-NSF materials. Reform recommendations of higher-level mathematical thought were beginning to guide the development of mathematical One theme common to the NCTM Standards and

to the recent changes in mathematics education is that \mathbf{r}

1989, p. 29).

calls for proposals that would create comprehensive instructional materials for the elementary, middle and high schools consistent with the calls for change in the *Curriculum and Evaluation Standards* [NCTM, 1989] (pp. 14). As a result of this project, *Everyday Mathematics* was developed as one of three comprehensive

instructional programs at the elementary grades funded by the NSF. Textbooks that are not funded by NSF are generally considered to be publisher-generated. By selecting NSF and non-NSF materials, I captured two contrasting perspectives from which these materials are produced.

Standards alignment. Mathematics standards documents provide recommendations for the content students learn. Because the textbook is the dominant tool used in classrooms (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr et al., 2008), many textbook companies profess to adhere to these standards documents. According to Reys and Reys (2006), most publishers claim to be

careful examination of materials is recommended to determine if this claim is actually true. The two textbooks I chose for analysis claim to be aligned to the newly developed CCSS (2010).

Principles and Standards for School Mathematics; however,

Overview of selected textbooks. For these three reasons (significant market share, NSF and non NSF funded materials, and standards alignment), I chose the $4th$ grade textbook from two series (with teacher editions): the 2011 edition of *enVision MATH* published by Pearson Education, Inc. and the 2012 third edition of books developed by the University of Chicago School Mathematics Project (UCSMP), funded by the National Science Foundation (NSF) titled *Everyday Mathematics, Common Core Edition*. Both of these textbooks are national versions and are not modified to fit the needs of any one specific state mathematics standards requirements. The textbook, *enVision MATH,* was not funded by NSF and is therefore labeled publisher-generated (Dingman, 2010).

enVision MATH. As the non-NSF funded program, Pearson (2011) posted the following statement on its website regarding the *enVision MATH* math program (www.pearsonschool.com: Scott Foresman-Addison Wesley enVision MATH © 2011): Pearson is making unprecedented levels of investment in new models for education and supporting key elements of the reform agenda: Common Core standards, college and career readiness, teacher effectiveness, school improvement, and custom solutions for schools and colleges (Pearson Education, Inc., 2011).

The materials provided by Pearson Education were one fourth grade *enVision MATH* Student Edition textbook and Lessons 1-20 Teacher Editions. The materials were obtained via email correspondences and phone communication directly from a Pearson Elementary Representative in the State of Florida. These materials were then analyzed and coded accordingly.

*Everyday Mathematics***.** Below is the language used by the NSF-funded series, UCSMP *Everyday Mathematics* posted on their website

(http://everydaymath.uchicago.edu/about/):

Everyday Mathematics is distinguished by its focus on real-life problem solving, balance between whole-class and self-directed learning, emphasis on communication, facilitation of school-family cooperation, and appropriate use of technology (UCSMP, Everyday Mathematics, para 2).

Everyday Mathematics with TIMMS international data findings. Carroll (1998) reported as follows:

Because of its research base, its international perspective, and its unique approach

Everyday Mathematics differs

substantially from other programs and has anticipated many of the concerns raised by the TIMMS report. In contrast to more traditional programs, in *Everyday Mathematics*

Exercises that require computation with digits do not require a student to construct a response other than digits. An example of an exercise that requires computation with digits specifically appears in Figure 1.

Find the sum of 37 and 28

37

+ 28

(Van deWalle, 2010).

Figure 1. Example of a computation specific problem type.

Also, I excluded exercises that led to a one-word answer. Exercises of this sort do not require the student to construct a response other than in a "one-word" form. An example of an exercise

Figure - wo

In addition, problem types that required numerical answers specifically in the form of digits written in standard or word form were excluded from the selection. Problems of this type do not require a student to construct a response other than in digit formation. An example of a problem type that requires an answer in numeric form, whether in standard or word form appears in Figure 3.

$Figure$

The final problem types excluded from the study were problems written in multiple-choice formats. These types of problems do not require a student to construct a response other than to identify the correct answer from a list of choices. An example of a problem type that is written in multiple choice format appears in Figure 4.

 $Figure$

Included items in mathematics textbook analysis*.* The criteria for the selection

of prompts aligned coney to the characteristics of \mathbf{s} . Cooney

et al. (2004) describe "open-ended" math as those that require students to

communicate their mathematical thinking,

Sample 1

How do you know 1/4 is greater than 1/5? Explain your thinking. Urquhart (2009)

Figure 5. A problem the term $\frac{1}{2}$ prompt.

In addition to the specific prompting items, the terms listed in Appendix B also have word *associations.* A word association is a term that is within the same family of words or meanings. An example of a word association can be described by a prompt that

ith the term

associated with a particular term (narrate) and was identified as a prompt that has the potential to facilitate a written response. Depending on the context of the prompt, the associations between words on the list in Appendix B to words in the prompt were also identified when the word was not listed explicitly. An example of a prompt that included

the association "we appears in Figure 6.

Sample 2

Write a sequence of actions occurring over time by relating the story of evolution of the abacus through ancient, middle, and modern times.

Urquhart (2009, p. 16)

Figure 6. Example problem type with a word *association*

In reviewing the *Everyday Mathematics* and *enVision MATH* textbooks for prompts that

miur>, I c] TJETBT1 0 0 1 1050.86709.2 Tm[(mod)-9(e)4(d the ta)4(sk as)] TJETBT1 0 0 1 1230.33709

sense based on the language that referred to number sense processes. The average number of prompts in the *number sense* category coded was 100% from Chapter 1.

*Framework revisions from question two***.** In the revised framework the category of *other* was added to the categories. Although the pilot study did not have any prompts coded as *other*, the process of identifying the language helped to determine that a category of this nature should be developed in the event the language was not indicative of the language within each of the content strand categories.

Question three. What types of vocabulary are used in the writing prompts in one $4th$ grade NSF-funded mathematics textbook and one publisher-generated textbook? Within the original framework the academic vocabulary categories were as follows: *domain specific vocabulary (DSV), general vocabulary (GV), meta-language,* and *symbols*. Words that had the potential to be coded as academic vocabulary based on the definition of each of the vocabulary categories were scanned in an Excel document comprised of four vocabulary word lists. If the exact term was not found in the lists, then any possible derivatives of the word were located. If a derivative of the word was still not located, an association of the word was acknowledged in order to determine what type of academic vocabulary the term *could* potentially be coded. Word associations assisted in determining if the term *should* be in a specific word list. If an association was made to a particular term not found in the word lists, it was coded under *words not on list.*

Once the words were coded in the *academic vocabulary* domain, I counted the total number of the words in each of the following ca() 13 (, m)-7(e)4(t4bT TJETBT1 0 0 1 409.47 657.1

71

a

example, the total words/symbols in the writing prompts that were coded as *academic vocabulary* for Chapter 1 was 72 out of 183 total words or 39%.

I then analyzed the total amount of words coded for each academic vocabulary category independently. The total count in each of the categories was then divided by the total number of words in order to determine which types of *academic vocabulary* were present. For example 37%, which was the majority of academic vocabulary coded from Chapter 1, was DSV. Furthermore, out of 72 total words identified as *academic vocabulary*, 7 of those words were not located on the a priori academic vocabulary lists. As a result, these words were placed in the *words not on list* category. Therefore, based on the definitions of the types of academic vocabulary, 10% of the words coded for Chapter 1 *should* be coded as academic vocabulary, but were not.

Framework revisions from question three. I made four revisions to the framework based on the analysis of the data from Question Three. The first revision was to change *special words* to *words not on list*. This domain name change appeared to be more representative of the status of the words. The second revision involved moving the dimension column nchaneiononctatus of the wo15 Tm14ol me cha

An analysis of the research questions across the framework dimensions provided for seven revisions to the framework. The first revision provided for additional dimensions to be added for purposes of calculating the average regarding the number of prompts. The second revision indicated that the category of *other* should be added to the content strands. The third revision changed the dimension of *special words* to *words not on list*. The fourth revision consisted of changing the symbols reference list to a more elementary mathematics *friendly* version. The fifth revision relocated the dimension of *words not on list* next to *academic vocabulary.* The sixth revision consisted of changing the name of *narrativizing and fictionalizing* math content in an *imaginary* or *real world* sense to *narrative* prompts in an *imaginary* or *real world* sense. The final revision consisted of changing *problem solving* to *generic* prompts.

The pilot study and the modification made to the framework, coupled with the research literature, provide an understanding of the framework presented.

Framework Dimensions

Modifications of the framework resulted in a framework with 10 dimensions: *number of writing prompts, number of exercises per page, statement of the prompt, content strand, academic vocabulary, words not on list, total number of words, type of prompt, teacher edition prompt support, student edition prompt location.* A table of the dimensions and code key are located in Appendix D. This framework of dimensions and code key was developed in the form of a matrix for the purposes of classification. (See Appendix A).

Furthermore, the framework dimensions were clustered according to themes in order to provide an understanding of the framework associations. For example, *number of* *writing prompts, number of exercises per page* and *student edition* were clustered as *page orientation.* The dimensions of *statement of the prompt, content strand, academic vocabulary, words not on list, total number of words, and type of prompt* were clustered as *prompt analysis*. The final dimension of *teacher edition prompt support* was identified as *prompt support.* In addition, the associations of the framework dimensions will assist in the organization of this section. (see Figure 10).

location of writing prompts within the textbook provided information regarding *where* the prompts were located. I also determined the trends in prompt location or language patterns within the section titles of each textbook by conducting a simple count of the various patterns within the language of the titles.

The following dimensions within the cluster of *prompt analysis* will be described further in the next section (see Figure 12).

Figure 12. Framework dimensions within the cluster of *prompt analysis.*

Statement of the Prompt*.* Within the *statement of the prompt* domain, the exact wording from the prompt was recorded. By recording the words in the prompt I was able to analyze the language that led to coding with the *content strand, academic vocabulary* and *type of prompt* dimensions.

Content strand*.* Within the *content strand* domain, the language within the writing prompt was coded to determine its alignment with a particular content strand/s

Table 1

Category	Topic
Number and Operations	Place value
	Base ten number system
	Whole numbers
	Negative numbers
	Decimals
	Fractions
	Percents
	Factors
	Multiplication of numbers
	Division of numbers
	Addition of numbers
	Subtraction of numbers
	Estimation of numbers

Topics within Number and Operations-Grades 3-5

An example of a prompt that would be coded in the category of Number &

Operations appears in Figure 13. This prompt would be coded in the Number &

Operations category because of the fraction symbol.

Figure 13. Example prompt coded Number & Operations.

Algebra.

representing and analyzing mathematical situations, patterns, relations, functions, structures, and quantitative relationships using algebraic symbols and models (NCTM, 2000). Table 2 presents the topics within the content strand of Algebra according to the *Principles and Standards* in Grades 3-5 (NCTM, 2000).

Table 2

Topics within Algebra-Grades 3-5

An example of a prompt that would be coded in the category of Algebra appears

in Figure 14. This prompt would be coded in the Algebra strand because of the unknown pattern.

What is the surface area of each tower of cubes (include bottom)? As the towers get taller, how does the surface area change?

Principles and Standards, (NCTM, 2000)

Figure 14. Example prompt coded Algebra.

Geometry. Geometry consists primarily of analyzing properties and relationships of geometric figures and shapes. Table 3 presents the topics within the content strand of Geometry according to the *Principles and Standards* in Grades 3-5 (NCTMsiBT /F2.ET onhapes. Write down everything you know and everything you can find out about this square.

Table 5

Topics within Data Analysis/Probability-Grades 3-5

Category	Topics
Data Analysis/Probability	Data
	Data set
	Categorical Data
	Numerical Data
	Observations
	Surveys
	Experiments
	Tables
	Graphs
	Line Plot

An example of a prompt that would be coded in the category of Data Analysis/Probability appears in Figure 17. This prompt would be coded in the Data Analysis/Probability category because of the probability reference in the prompt.

If two coins are tossed, what could happen?

(Sullivan & Lilburn, 2002)

Figure 17. Example prompt coded Data Analysis/Probability.

Other. Based on the pilot study, the category of *other* was developed for prompts that could not be categorized within the five content strand categories. If the language within the writing prompts was not indicative of the language within the content strands then the prompt was coded under the category of *other*. An example of a prompt coded in the category of *other* appears in Figure 18. This prompt would be coded as *other* because the language within the prompts is not indicative of the language associated to the mathematics topics indicated in Tables 1-5.

Do you know anyone who has visited or lived in this country? If so, ask that person for an interview. Read about the country's customs and about interesting places to visit there. Use encyclopedias, travel books, the travel section of a newspaper, or library books. Try to get brochures from a travel agent. Then describe below some interesting things you have learned about this country.

(Everyday Mathematics 4 th Grade Student Journal, 2010)

Figure 18. Example prompt coded Other.

An analysis of the language within the prompt assisted in determining which content strands had the majority of writing prompts. In addition, an analysis of the prompt language also provided information regarding the type of academic vocabulary identified within the writing prompt.

-specific vocabulary, (2) general vocabulary, (3) literary

vocabulary, (4) meta -language , and (5) symbols. A modified versn>x of the Bauma nn and Graves (2010) word classi fication scheme (see Appendix E) was used as a guide for

primary source for word classification in the category of GV. If the word was polysemous, with different definitions being relevant to different domains, then the word was coded as GV. An example of a wording in a prompt that was coded in the category of GV is underlined and appears in Figure 20. The word area was found in the Coxhead

What is the surface area of each tower of cubes
fraction bar $($), and two digits $(1, 2)$ this particular symbol was analyzed as four symbols. Therefore the following symbols in the prompt were calculated as six symbols total

 $-, 1, \backslash, 2)$ $\overline{(\ }$

Write some different stories about $\underline{3} \pm -?$

(Sullivan and Lilburn, 2002)

Figure 22. Example code for Symbols.

Words not on list. Within the words not on list domain, I recorded words that were not identified in the academic vocabulary a priori word lists but *should* be according to the definitions of the academic vocabulary categories. Because the framework was developed from the most extant work on typologies of academic vocabulary by the Baumann and Graves (2010) word classification system, words that specifically met the criteria of the categories were analyzed and coded. However, if a word was not listed in the academic vocabulary word lists, it was coded in the dimension words not on listeo.2(ords n)8(ot d92n

Analysis

I reviewed 100% of the numbered or lettered

 $_{\rm CO-}$

Reliability training. During my first session, I trained both co-raters using the pilot study as my guide. Additionally, a codebook was used as a reference for selection of the prompts and coding the prompts across the framework dimensions (see Appendix K). After the training session, I gave the co-raters the same textbook used in the pilot study and asked them to code the chapter using the framework. In order to determine the reliability of my prompt selection, the raters used the criteria of terms provided in Appendix B and in the coded book (see Appendix K). After the selection of prompts, the co-raters and I compared our coding and discussed any discrepancies. After the writing prompts were discussed, a blank framework in the form of an excel document was given to each rater. Next the co-raters rated the prompts along the dimensions of *content strand, academic vocabulary, type of prompt* and *teacher edition*. The co-raters used Appendices E-I for academic vocabulary with the codebook as a reference tool to code across the dimensions. Once the coding was complete the co-raters and I compared our coding across the dimensions and found consistency in our selections. After the training using the pilot study, we felt there was a common understanding of the analytical framework and the co-raters were ready to code on their own.

Lessons coded in the textbook's section and content areas that sections are the textbook in the textbook's sections are the textbook's sections and content areas that the textbook's sections are the textbook's sections a

had a numbered or lettered exercise. The pages that consisted of a numbered or lettered exercise were titled *readable pages* for purposes of this study. Pages that were not coded did not have a numbered or lettered exercise on the page. The two co-raters reviewed 10% of the readable pages in order to assess agreement on the prompts to be included for analysis. I developed an itemization of the number of exercises within each chapter in order to provide ease of selection for the 10% of readable pages to be co-coded. Based

98

Table 7

Final Decision. As noted in the table above, 37 unique prompts were identified across all three co-coders and 34 were included for analysis. After discussion, the cocoders and I collectively decided to eliminate three tasks as writing prompts because of the nature of the constructed response. For example, if the prompt could be answered in a one word response, the prompt was not included for final coding. In all three of the eliminated prompts, the prompt affordance was in the form of a one word answer. The following is a prompt that was eliminated based on the affordance of a one-word answer:

Of the 32 prompts I coded individually, 100% of those prompts were included in the final count of 34 prompts agreed upon for analysis. The additional two prompts were

identified by my co-coders. Therefore, the reliability of content strand selection for the enVision MATH was calculated using the final number of prompts as the referent⁵.

Reliability of prompt selection in Everyday Mathematics. After reviewing the coding it was determined that all three coders had 100% of the coding consistent with one another. For example, of the 21 prompts coded in both lessons, 100% of those prompts were the same prompts among both co-coders and me. Therefore, there were no prompts identified by one only one rater and there was 100% baseline agreement.

There are several reasons that might explain why the agreement was higher in *Everyday Mathematics* than *enVision MATH*. First, this textbook was coded second and the previous coding may have made the prompt selection easier. Second, the layout of the *Everyday Mathematics* textbook has fewer tasks per page, sometimes having only one or two() lesbBTefhe agreement was hinathose p(twwa)6(s 100)-10(%)3(ba)4(se)3(li)-3(ne)4(3681.58 Tr *number of exercises per page, student edition prompt location* and *academic vocabulary total.* Additionally, I decided not to check-code the *words not on list* dimension since this section was used as a category to place words that each of the raters *believed* to be academic vocabulary but could not locate in the a priori word lists. Therefore, the dimensions that were less obvious regarding coding assignment were *content strand,*

between the researcher and the co-raters. As the researcher, if the language within the prompt was located in more than one content strand, then I coded it accordingly into multiple content strands. Once I made the co-raters aware of the language in the prompt and how that language assisted my decision, they agreed regarding my codes. For example, the following prompt is an example of a prompt coded in th

Table 9

Percentage of Agreement of Coding for Content Strand for Everyday Mathematics

Before Discussion

was completed in this section for each textbook, discussions regarding words omitted and

During our discussion of the words missed, I simply had to show Rater 1 and Rater 2 where the words were located on the a priori word lists. For the omitted words, Rater 1 and Rater 2 had two word associations that were placed in the *words not on list* category after our discussion. For example, Rater 2 coded the term *translation* as *domain specific* because *slide transformation* was located in the *domain specific* list. In addition, Rater 2 also understood that a *slide transformation* is a type of *translation*. However, because the words are associated and not derivatives, the term was placed in the *words not on list* category.

Reliability of coding of academic vocabulary for Everyday Mathematics. The reliability of academic vocabulary selection for *Everyday Mathematics* was calculated using the final number of words agreed upon as the referent⁹. As the researcher, I missed 10 words word due to an oversight that Rater 1 and Rater 2 had located in the word lists. Before discussion I had 91% of the codes in agreement with the final decision. The words that I missed were commonly used so they resulted in the same word being missed across multiple writing prompts. Before discussion, Rater 1 had 84% of the codes determined in our final decision and Rater 2 had 82% of the codes in agreement with the final decision. Similar to our previous discussions based on words missed and words omitted, Rater 1 and Rater 2 had changed the codes to reflect 100% agreement (see Table 11).

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 9^9 I was the only coder for 90% of the textbook. It is important to determine how close my codes were to the final decision so that the results from my coding may be the same if coded by others.

Table 11

Percent of Agreement of Coding for Academic Vocabulary for Everyday Mathematics

Before Discussion

Note. Percent is determined by no of words per coder/final number of words $(n=101)$.

Reliability of coding of type of prompt. Based on the research in mathematics writing prompt types (Burns, 2004; Dougherty, 1996; Urquhart, 2009; Whitin & Whitin, 2000), the codes for *Type of Prompt* include *narrative*, *affective* and *generic* problem. The percentage of agreement in this domain was 100% among the researcher, Rater 1 and Rater 2 for both *enVision MATH* and *Everyday Mathematics* textbooks. The checkcoding system indicated that the researcher and the co-raters coded 100% of the prompts the same. If a writing prompt did not require the student to answer in the form of an affective/attitude response, nor did it require the students to write a story, then the writing prompt was coded as *generic*. The high reliability in this domain may be a result from the high percentage (99%) of the writing prompts in both textbooks coded in the *generic*

Reliability of coding of teacher edition for Everyday Mathematics. The coding of the teacher edition for this textbook was based on writing prompt support. The reliability of teacher edition for *Everyday Mathematics* was calculated using the final number of prompts agreed upon as the referent¹¹. As the researcher, I had 100% of the codes in agreement with the final decision. Similarly, Rater 1 also had 100% of the codes in agreement with the final decision. Before discussion Rater 2 had 75% of prompts in agreement with the final decision. Based on discussion it was determined that an oversight occurred with Rater 2. After discussion, Rater 2 agreed with the researcher and Rater 1 to reflect 100% agreement of the final decision (see Table 13).

Table 13

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Percent of Agreement of Coding for Teacher Edition for Everyday Mathematics

Co-coders	No. of Academic Vocabulary	%
Researcher	21	100
Rater 1	21	100
Rater ₂	16	76

Before Discussion

Note. Percent is determined by no of words per coder/final number of prompts $(n=21)$.

 11 I was the only coder for 90% of the textbook. It is important to determine how close my codes were to the final decision so that the results from my coding may be the same if coded by others.

Summary of Reliability of Framework Dimensions

The reliability of the coding within the framework led to co-coding in five of the dimensions for each textbook investigated: *statement of the prompt, content strand, academic vocabulary, type of prompt, and teacher edition prompt support*. Most discrepancies in coding were based on an oversight and were adjusted to reflect 100% of the final agreement. The training session integrating the codebook (see Appendix K) and collaborative discussions were important in achieving the reliability.

Sources of Influence

There are two sources of influence that have the potential to affect the reliability of my study. The first source of influence in the study is my bias interfering in training my co-raters. In order to reduce this training bias, I selected two raters instead of one to assist in coding the data within each of the dimensions. In an effort to obtain at least 80% agreement, discussions with additional modifications to the framework categories were addressed. My second source of influence was how the textbooks were chosen for the study. Within my lite

Chapter 4: Results

Writing Prompts

To determine how many of the total exercises were writing prompts, I isolated the student exercises that were identified with a number or a letter. If the exercise afforded the opportunity of a response using one or more sentences, it was coded as a prompt for written response. For example, the following prompt from the *enVision MATH* textbook was coded as a writing prompt:

How does using commas to separate periods help you read large numbers?

From the 20 lessons analyzed in the *enVision MATH* textbook, 323 tasks were coded as writing prompts out of 2,481 exercises (13%). From the 14 lessons analyzed in *Everyday Mathematics*, 140 tasks were coded as writing prompts out of 704 exercises (20%). Table 15 shows a description of the tasks analyzed and coded as writing prompts within both textbooks.

Table 15

Exercises and Prompts within the enVision MATH and Everyday Mathematics Textbooks.

Although *enVision MATH* (*N=*323) included more writing prompts than *Everyday*

If a prompt had language that was used and identified within two content strands, the prompt was coded in both content strands. For example, the following prompt was coded in both the *number sense* and *measurement* categories:

How many hundredths are in one-tenth? Explain using pennies and a dime.

was coded as *number sense* (see Table 1). In addition, the language of "pennies" and "dimes" was coded as *measurement* (see Table

4). This prompt was located in the lesson section titled Using Money to Understand Decimals. In total, 55 *enVision MATH* prompts were dually coded and 57 *Everyday Mathematics* prompts were dually coded.

Across the content strands both textbooks included approximately the same percentage of prompts in Geometry, Measurement, Algebra, and Data Analysis. The exceptions were: *number sense* and *other*. Both the *enVision MATH* textbook and *Everyday Mathematics* textbook had the largest percentages of prompts recorded in the *number sense* category. However, there were differences in the percentages recorded for each textbook that may be explained by the fact that 21% of *Everyday Mathematics* prompts were coded in the content strand of *other* and *enVision Math* had 0% coded in this category. Prompts coded in the section of *other* did not have any mathematical content language needed to identify a content strand category. Within the *Everyday Mathematics* textbook, these prompts were identified in lessons titled *My Country Notes.* These prompts dealt with particular questions associated with countries around the world.

Content strand and textbook. As indicated in Table 16, both of the textbooks had the highest percentage of writing prompts coded as *number sense* tasks. However, the category of *other* had the largest percent *difference* between the two series. Only the

118

Everyday Mathematics Enm[(E7(x Mattbook had wda)-iting)7(prompts code)6(d ada)sETBT1 0 0 1 f1 0

Figure 24. Percentage of prompts within each content strand of *enVision MATH* and *Everyday Mathematics* (EM) textbooks.

Academic Vocabulary

The third research question related to the type of vocabulary coded within the

prompt uses bolded font to indicate the vocabulary identified and coded as *general vocabulary*:

> If you buy an **item** that costs \$8.32, why would you pay with one \$10 bill, 3 dimes, and 2 pennies?

Third, words coded as *meta-language* usually described a process (see Appendix H and

I). The following prompt uses bolded font to indicate the vocabulary identified and coded as *meta-language:*

Why

second highest average of 33% coded as *domain specific vocabulary* and 27% as *meta-*

language. General vocabulary had the lowest average of 5% between the two textbooks.

Table 18 provides detailed information regarding these percentages.

Table 18

Type of Academic Vocabulary within the Writing Prompts in the enVision MATH and Everyday Mathematics (EM) Textbooks.

		enVision MATH	EМ		enVision Math & EM
Type of Academic Vocabulary	\boldsymbol{n}	$\%$	\boldsymbol{n}	$\%$	Total %
Domain Specific Vocabulary 730		34	259	31	33
General Vocabulary	117	5	42		

language had the largest percentage difference (6%) between the two textbooks. The percentage of *general vocabulary* was not only the same for both textbooks but also the lowest percentage in each textbook.

Figure 25. Percentage of academic vocabulary within the writing prompts in the *enVision MATH* and *Everyday Mathematics* (EM) textbook. *Note.* DSV = Domain Specific Vocabulary; GV = General Vocabulary; ML = Metalanguage; $S =$ Symbols

Included in the percentages for *Academic Vocabulary* were derivatives. For

example if the word *explain* was located in the prompt, the word was coded as meta-

language since *explanation* is the derivative found in the meta-language word list. A total

of 440 words were identified as derivatives of the word lists.

Academic vocabulary and words per prompt. In total, 2,157 out of the 5,748

total words within the 323 prompts located in the *enVision MATH* textbook were coded

as academic vocabulary. Therefore, an average o 0 0 177E7.] TJETBT1 0 0 10409.81 120.14 Tm[()] TJI

coded prompts. Therefore, an average of 18 words per prompt was indicated. Because an average of 7 words per prompt were coded as academic vocabulary out of the 18 average words per prompt, approximately 37% of the words within the prompt were coded as academic vocabulary for *enVision MATH* (see Table 19).

Similarly, 843 words out of the 3,211 total words within the 140 prompts located in the *Everyday Mathematics* textbook were coded as academic vocabulary. Therefore an average of 6.02 academic vocabulary words per prompt was determined (see Table 17). In addition, 3,211 total words were counted within the 140 coded prompts. Therefore, an average of 23 words per prompt was indicated. Because an average of 6 words per prompt were coded as academic vocabulary out of the 23 average

language, and *symbols* but were not located on the a priori academic vocabulary word lists. Once identified as *academic vocabulary*, the words were then scanned in the academic vocabulary word lists (see Appendix F-J) for purposes of categorizing. If the word or the derivative of the word was not located in one of the vocabulary word lists, it was placed in the *words not on list* category. Overall, within the *enVision MATH* and *Everyday Mathematics* textbooks1,679 words were placed in the *words not on list* category. Although many of the words were duplicates, they were labeled in the *words not on list* category as DSV, GV, or ML by definition of the academic vocabulary categories (see Appendix A). For example, *pennies* and *dimes* were located on more than one occasion and coded as DSV by association to the term *money* in the DSV word list. The number of each of the words that *could* potentially be in the a priori academic vocabulary word lists can be found in Table 20.

Table 20

\bullet		
Academic Vocabulary Category	n	
Domain specific vocabulary	591	
General vocabulary	296	
Meta-language	792	
Total	1679	

Words Not on List Within the Writing Prompts in the enVision MATH and Everyday Mathematics (EM) Textbooks.

Type of Prompt

The final research question related to the type of prompt located within each textbook. The language used within the prompt had the potential to determine the type of prompt: affective or expository. Affective prompts (Baxter et al., 2007; Shield $\&$ Galbraith, 1998) are prompts that intend to elicit opinions or feelings. Because *enVision MATH* did not have any prompts coded as *affective,* the following prompt from *Everyday Mathematics* is used as an example of an *affective* prompt. The language used within the prompt required a constructed response of an opinion or feeling:

What are some things you have enjoyed on the World Tour? (p. 325). Expository responses are responses that do not involve feelings or opinions but more problem-solving or explaining a process in mathematics (Baxter, Woodward & Olson, 2005). I used the category, *generic,* to code writing prompts that aligned with the expository definition. The two prompts below were coded as *generic*:

1) Explain why the value of 5 in 5,264 is 5,000 (*enVision MATH*, p. 4).

2) Feng said the name of this ange used withi used withi

literature (Burns, 2004; Whitin & Whitin, 2000). The following math prompt was noted
category of *content and process* prompts led to a deeper investigation of the language

Gina pays for an item that costs \$6.23 with a \$10 bill. **What is** the least number of coins and bills she could get as change? **Explain.**

These findings of a dual stem indicate the complexity students may encounter when having to answer both a question *while* providing an explanation to a command.

The analysis of the *type of question* indicates there were 13 variations of *how* questions, 11 variations of *why* questions, 9 variations of *what* questions and 2 variations of *when* questions. In the *type of command* category, findings indicate there were 3 variations of *describe* commands, 7 variations of *explain* commands, 7 variations of *construct* commands using *write*, *make* and *give* as stem words (see Appendix M).

A further analysis of the *types of question* category indicate the variations of *how* were the most common form of question stem. The second most common form of question stem were the variations of *why*. Even though the percentages were lower in the categories of *what* and *when*, students were also encouraged to construct responses to these forms of questions (see Table 21). In the *types of command* category, the most common command required the student to *explain* a response. The second most common command required the student to respond by the use of a construction to the command words of *write*, *give* and *make* (see Table 22).

Table 22

Question Stems	n
How	111
Why	64
What	26
When	2
Total	203

Number of Mathematical Prompt Stems of Generic Category

Table 22 (*continued*)

Command Stems

The results of the analysis of prompt stems indicated a multitude of question and command stem variations for students to decipher in order to construct a response. As the students construct a response to mathematical prompts, they must also consider processes such as problem solving, reasoning and proof, communication, connections and representations flexibly while utilizing mathematical vocabulary and symbols. Strategically, problem solving strategies such as pattern recognition, working backwards, guess and test, experimentation/simulation, reduction/expansion, organized listing/exhaustive listing, logical deduction, and divide and conquer (Krulik & Rudnick, 1995) should also be implemented during the construction process of the prompt. Furthermore, mathematical process and problem solving strategies should also incorporate the structures of writing during composition. Fang and Schleppegrell (2010) note literacy structures of listing, description, explanation, sequence, compare/contrast, cause/effect, and problem/solution are encouraged in writing and reading within the content areas. The projected constructed response of the generic prompt should utilize mathematical process standards while integrating mathematical strategies and literacy structures. For example, in order for a student to construct a response to a problem, many of the problem solving processes can be used simultaneously (such as reasoning and

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strategies such as *pattern recognition* and *logical deduction* can also be utilized while implementing the literacy structures of *descriptions and sequences*. This interwoven, recursive process of the complex nature of integrating writing in the mathematics content area can be found in the form of a model in Appendix M.

Affective Prompt

Only *Everyday Mathematics* had prompts coded within the *affective* category. These types of prompts require students to construct an answer that is associated with an attitude or feeling about mathematics. According to Dougherty (1996), these types of prompts provide a more holistic view of how students view mathematics. The following prompt was coded as *affective* from the *Everyday Mathematics* textbook:

What are some things you have enjoyed on the World Tour? The prompts coded as *affective* were located in a section titled *World Tour.* This section infused the content area of social studies within the *Everyday Mathematics* student textbook. Although words specific to the domain of mathematics were not located in these prompts, the prompts were coded as *affective* because they included language indicating a feeling or attitude. Additionally, these prompts were located in the student edition of the *Everyday Mathematics* textbook.

Narrative Prompt

Everyday Mathematics also had the majority of prompts coded *narrative.* These prompts were coded in a lesson sect and were related to and were related to touring a country. More specifically, the

Therefore, all of the prompts coded in this section were further

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classified as *real world* and not *imaginary*. In addition, only one prompt (<1%) was located in this category of the framework within the *enVision MATH* textbook. Figure 26 provides more information regarding the percentages calculated within this category of the framework.

Figure 26. Percentage of the types of prompts in the *enVision MATH* and *Everyday Mathematics* (EM) textbook. *Note.* $G =$ Generic; $A =$ Affective; $N =$ Narrative

Other Framework Categories

Although the framework was designed specifically to align to the research questions (see Appendix A) by examining the nature of writing in two mathematics textbooks, the additional categories of *teacher edition* and *student edition* assisted in providing another layer of analysis regarding the prompts. Exploration of the teacher edition enhanced the research questions by providing information on how the writing prompts were supported from an instructional standpoint. In addition, an examination of

the writing prompt location in the student edition also had instructional implications. Information regarding the sections and subsections and additional subsections of where the prompts were located in the student edition provided information of how *enVision MATH* and *Everyday Mathematics* situate writing in mathematics.

and *Everyday Mathematics* have support for over 90% of the writing prompts (see Figure 27).

Figure 27. Percentage of types of support for the prompts within the Teacher Edition in the *enVision MATH* and *Everyday Mathematics* (EM) textbooks. *Note.* Su= Support (only); Sa=Sample (only) ; SS=Support with Sample; N=No Support or Sample

Student edition. The domain section of *student edition* in the framework contained three sections titled: *section, sub-section,* and *additional sub-section.* The layout of the student editions of both textbooks varied greatly. Although the lesson numbers were close in range (*N=*20 and *N=*13) the number of section titles within these lessons differed to a great extent.

Student Edition and textbook. Upon analysis of the three categories within the dimension of *Student Edition*, the *enVision MATH* textbook had more coding in each of the categories than *Everyday Mathematics.* Because there were limited sub-sections or additional sub-sections located within the *Everyday Mathematics* textbook, the language

was too complex and varied to analyze for patterns. Because each topic section had a different title, the language analyzed within the title provided no pattern for analysis; most every topic section title had a different heading using different language in *sections, subsections,* and *additional subsections*. (see Appendix N). Additionally, the language within the section titles of the *Everyday Mathematics* student textbook contained words specific to mathematics. Therefore a simple calculation of the amount of DSV was conducted within the sections of each lesson. Approximately 101 words were calculated to be DSV in *Everyday Mathematics* section titles of the student edition and 11 words in the section titles of the *enVision MATH* textbook.

Conversely, only the *enVision MATH* textbook provided data in this domain across all three categories for patterns in language in the section titles. Since there are titles in the sections, sub-sections and additional sub-sections, the analysis of the language within the titles of these categories revealed patterning. This patterning found in the language of the section titles allowed for a visual representation in the form of a graph to be developed. Figure 28 provides an example of *section*, *sub-section* and *additional sub-section* titles of the prompt location within the student edition of *enVision MATH*.

Figure 28. Example 10 Figure 28. Example of the "section titles" for a writing prompt within a student edition page.

As indicated in Figure 29, the largest percentage of writing prompts was located in the sections of *guided practice* and *independent practice.* The lowest percentages are in the *algebra, enrichment,* and *practice* sections.

Figure 29. **Percentage of prompts with Student Edition Stude** *enVision MATH* textbook.

Table 24

	enVision MATH	
Category Language	n	
Writing	94	
Understand	117	
Explain	89	
Reasoning	53	
Problem/Problem Solving	100	
Total N of Words	453	

Number of Category Language within the Student Edition for enVision MATH

Cross Analysis

As revealed in the previous sections within this chapter, the analysis of prompts within the content strands revealed trends within the framework dimensions. As a result, I determined an additional analysis across the dimensions was necessary to provide a context for the findings of the individual strands. Therefore, using a matrix, I cross analyzed the results from my analysis of *content strand* categories (i.e., number sense, geometry, measurement, algebra, data analysis and other*)* with (1) the categories of *academic vocabulary* (i.e., domain specific vocabulary, general vocabulary, metalanguage, and symbols), (2) *type of prompt* (generic, affective, and narrative) and (3) *teacher edition information* (i.e., support, sample, support with sample, and no support or sample). In order to determine if any patterns were revealed, simple calculations, using the data from each of the categories were used during the cross analysis. The findings from the matrix analysis are discussed in the following section.

Cross Analysis within *enVision MATH.*

Content Strand and Academic Vocabulary. Within the content strand of *number sense*, the matrix analysis revealed that symbols were the most frequent form of academic vocabulary used in *number sense* prompts. Approximately 43% of the academic vocabulary coded in *number sense* was comprised of *symbols*. Within the *geometry* content strand, the largest percentage of academic vocabulary was *domain specific vocabulary*. Approximately 54% of the academic vocabulary in geometry was classified as *domain specific vocabulary.* An analysis of the content strand of *measurement* was similar to *number sense* in that the largest percentage of academic vocabulary was coded as *symbols*. Within the *algebra* content strand, 33% of the academic vocabulary was coded as *symbols* and 35% was coded as *domain-specific vocabulary*. Within the content strand of *data/probability* the largest percentage (35%) was coded as *domain specific vocabulary* (see Table 25).

Content Strand and Type of Prompt. Findings in the content strand of *number sense* indicated 99% of prompts were categorized as *generic* prompts. Less than 1% of prompts in number sense were located in the *narrative* category. Furthermore, results indicated that 100% of the prompts in *geometry, measurement, algebra,* and *data/probability* were coded as *generic* prompts. There were no prompts coded as *affective* within the *enVision MATH* 4th grade textbook (see Table 25).

Content Strand and Teacher Edition Prompt Support. The cross analysis of content strand with teacher edition revealed the most common form of support for *number sense* prompts was both *sample* and *support with a sample*. Approximately 49% of the support was in the form of a *sample* and 48% was in the form of *support with a*

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sample. The largest percent of teacher-edition prompt support for *geometry, measurement, algebra* and *data/probability* was coded as *support with a sample* (see Table 25).

Table 25 (*continued*)

Cross Analysis within *Everyday Mathematics.*

Content Strand and Academic Vocabulary. Within the content strand of *number sense,* the matrix analysis revealed that *symbols* were the most frequent form of academic vocabulary coded in the *number sense* prompts. Approximately 39% of the academic vocabulary coded in *number sense* was comprised of *symbols*. Within the *geometry* content strand, the largest percentage of academic vocabulary was *domain specific vocabulary.* Approximately 43% of the academic vocabulary in geometry was coded as *domain specific*. An analysis of the content strand of *measurement* was similar to *number sense* in that the largest percentage of academic vocabulary was coded as symbols. Approximately 45% of the academic vocabulary in *measurement* was coded as *symbols.* The *algebra* content strand was similar to *number sense* in that the largest percentage of academic vocabulary was coded as *domain specific*. Approximately 45% of the words coded in the *algebra* strand were coded as *domain specific*. Within the *data analysis/probability* content strand, 39% were coded as domain specific and 36% were coded as *meta-language*. Therefore the *data analysis/probability* were only separated by a 3% difference. The final category of *other* indicates that 72% of the prompts were coded as *meta-language* (see Table 26).

Content Strand and Type of Prompt. Findings in the content strand of *number sense* indicated 97% of prompts are categorized as *generic*

*Content Strand and Teacher Edition Prompt Support***.** The cross analysis of

Table 26

Table 26 (*continued*)

Everyday Mathematics	Academic Vocabulary			Type of Prompt			Teacher Edition				
Content Strand	DSV	GV	ML	- S	G	\mathbf{A}	$\mathbf N$	Su	Sa	SS	$\mathbf N$
Data/Probability		$(n=77)$			$(n=15)$			$(n=15)$			
$\%$	39	6	36	18	93	θ					

Chapter 5: Discussion

Draper, 2001; Pugalee 2004, 2005; Senk & Thompson, 2003; Shulman 1986). More

selected two textbooks with different educational philosophies in order to understand how writing was incorporated in NSF-funded and publisher-generated textbook curricula.

I developed an analytic framework using 10 dimensions with respective subcategories based on (1) Principles and Standards for School Mathematics content strands, (2) Baumann and Graves (2010) classification scheme of academic vocab

Based on my analysis of these two textbooks, there are six major findings related to my research questions and these are explicitly discussed in the following sections.

1. The Questionable Focus on Number Sense

The NCTM *Principles and Standards for School Mathematics* (NCTM, 2000) indicate the following discrete content strands: *number sense, geometry, measurement, algebra, data analysis.* To categorize writing prompts by content strand, I used the language in the lesson title and within the prompt as well as the topic language listed in NCTM (2000) content strands (see Tables 1-5).

Standards documents and state assessments*.* The high percentage of prompts coded in the *number sense* strand aligns to the National Assessment of Educational Progress (NAEP, 2005) framework. The NAEP framework, which was developed to

of writing prompts instead of the majority of prompts located in the strand of number sense?

(PSSM, Executive Summary, 2000, p. 4)

Figure 30*.* Emphasis of the content standards across the grade bands.

Number sense as constrained skill*.* If the reason for the emphasis on number sense is related to standards and textbooks, then the reason is not a mathematical one given the need for students to develop mathematical thinking in geometry and algebra (Battista, 2007; Moses & Cobb, 2001a; Paul, 2003;). For example, according to Clements and Sarama (2007) early childhood and primary grades *number and operations* is arguably the most important area in mathematics learning and one of the best developed areas in mathematics research (p. 466). However these claims are only relevant to children in *early childhood* and *primary grades.* Although number sense in the middle and high school grades encompasses important content such as whole numbers, fractions, decimals, percents, proportions, and integers and number theory (NCTM, 2000), students in the intermediate grades are also encouraged to develop mathematical skills and

strategies in other content areas such as algebra. This focus on other content strands is in preparation for future success in mathematics. For example, *algebra* appears to have

significant importance and has been identified as the "Gate-Keeper" for future success

beyond the early grades school mathematics curriculum (Stinson, 2004). Additionally, Moses and Cobb (2001a) noted that the content associated with Algebra possesses gatekeeping power for college mathematics.

In support of this finding (as cited in Stinson 2004, p. 11) Algebra is the

do not take it in middle school (U.S. Department of Education, 1997, p. 5-6). Furthermore, students who enrolled in algebra as eighth-graders were more likely to reach advanced mathematics courses (e.g., algebra 3, trigonometry, calculus). Additionally students who enrolled in algebra as eighth graders and completed an advanced math course during high school were more likely to apply to a four year college than those eighth-grade students who did not enroll in algebra as eighthgraders but who also completed an advanced math course during high school (U.S. Department of Education, 1999, p. 1-2).

The continued emphasis on number sense through the intermediate grades appears to be analogous to the inappropriate practice of focusing on lower-level skills in the field In general, letter knowledge, phonics,

and concepts of print are highly constrained, phonemic awareness and oral reading

(2005, p. 187). These skills are that "skills such as alphabet knowledge" in that "skills" in that "ski are most related to decoding in early childhood, whereas unconstrained skills such as

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Although phonics is an integral part of emergent reading, the continued instruction of phonics can potentially hinder the analysis of reading comprehension skills (Dennis, 2012, Dennis & Parker, 2010; Paris, 2005). Could this analogy to constrained skills in literacy align to the heavy focus of number-sense instruction in the intermediate grades and potentially constrain mathematical skills such as measurement, algebra, and

communicate by way of reasoning, problem solving, and justifying thinking while also utilizing the process skills of connecting and representations? As a potential solution and as an attempt to provide more of a balance in the types of writing tasks across content strands,

area may be seen as a type of constraint for mathematical thinking in other content areas. An attempt to address this concern is the modification of writing prompts in mathematics textbooks to include domain specific vocabulary associated with other mathematics content areas such as geometry, algebra, etc. This modification of prompts could provide more of a balance to facilitate writing within other mathematical content areas. However, the revising of prompts would require the implementation of educational training programs. The implications for teacher educators and professional development is to assist preservice and inservice teachers in identifying where the writing prompts are located in the curriculum and then to modify or develop further prompts for instruction in the different content strands. Regardless of the textbook scope and sequence, teachers can locate writing prompts in the lesson and modify the language and vocabulary to meet the expectations of upcoming content if there are no writing prompts within the lesson *or* if the number of writing prompts are minimal. This information has the potential to provide insight to the field of mathematics by investigating how this type of knowledge could assist preservice and inservice teachers in identifying prompts that are suitable for their instructional goals.

Content strand summary. The need for students to encounter writing prompts across content areas is an important consideration for textbook publishing companies, teacher education programs and professional development. First, writing provides students with an opport

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students could explain a process such as reasoning while utilizing the vocabulary needed to construct a response:

Why do you think a square can also be called a rectangle, but a rectangle cannot

be called a square?

An answer to this prompt could provide teachers with evidence of studen understanding because their writing offers teachers a window into their thinking (Sowder,

2007)

domains. *Meta-language* was the term used to describe words associated with processes, structures, or concepts commonly included in content area texts. *Symbols* was the term for mathematical notation. The fifth category of Literary Vocabulary was not relevant to my study and therefore was not used in the classification scheme.

An additional analysis across the dimensions of the framework was conducted to provide a context for the findings of the individual content strands within the framework. The use of a matrix assisted in the cross analysis of the *content strand* categories (i.e., *number sense, geometry, measurement, algebra, data analysis* and *other)*

important to note the types of vocabulary most often encountered within these prompts. The high percentage of academic vocabulary containing symbols in the writing prompts aligned to the notion of *symbols* being the hallmark of mathematics (Thompson & Rubenste

both textbooks were coded as highly technical complex vocabulary such as symbols and domain words

Derivatives*.* During the co-rating session of this study, the co-raters missed a few words because the co-raters were not familiar with word derivatives and associations for certain academic vocabulary. For example, the term *multiplication* is in the DSV list. However this term has derivatives of *multiply, multiplied, multiplier, multiple*, etc. If the term multiply was encountered, it should be coded as DSV because it is a derivative of multiplication. However, my co-raters missed these terms. Due to my familiarity with the lists, I was able to help my co-raters identify some of the derivatives of terms they missed.

Associated Terms*.* Additionally, words that were not only derivatives of academic vocabulary but *associated* with academic vocabulary were not included in these lists. As a result, many terms that should have been coded were labeled as *words not on list*. For example, the term *day* is found in the DSV list. However, the actual days of the week, *Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday*, are not located in the *DSV* category. Therefore because of the word structure, these word associations were coded into the *words not on list* word list.

The words included in the *words not on list* dimension *should* be in the a priori word lists but were not. For example, the terms *gallon*, *dollar*, *milliliter*, and *trapezoid* are vocabulary that *should* be included in the DSV list but were not. Furthermore, the word lists including process words in the *meta-language* category should also be updated. This category had the majority of words indicated in the *words not on list* category. The words *answer* and *know* are not in the meta-language word list but were located on multiple counts in the writing prompts. For example, the word *answer* was located 71 times and the word *know* was located 50 times within the writing prompts. These words

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mathematical literacy per content strand is encouraged due to the ambiguity of the mathematical language used in the prompts.

4. Ambiguity of Prompts

I used the categories of *affective, narrative,* and *generic* to code the types of prompts textbook publishers utilized in two mathematics textbooks. An *affective* prompt is one that has language that elicits an opinion, feeling or attitude towards math (Baxter et al., 2007, Shield & Galbraith, 1998). A *narrative* writing prompt requests the writer to construct an answer that displays math content in imaginary or real world sense. Narrative math content is encouraged in the field of mathematics as an instructional tool and supported through the use of children's literature (B urns, 2004; Rubenstein \mathbb{R}

Thompson, 2002; Shiro, 1997, Thompson, 1997; Whitin & Whitin, 2000). The final category of *generic* prompt is inclusive of all of the prompts that were *not* coded as *affective* or *narrative*.

Generic prompts. The *generic* prompt category accounted for 93% of total prompts within both textbooks. According to the research in mathematics writing, these generic prompts were classified as either *content* or *process* prompts (Dougherty, 1996; Urquhart, 2009). For example, I coded the following *enVision MATH* prompt as *generic* as it required the students to utilize both *processes* and *content* in order to construct a response:

Can a circle and a square ever be congruent? Why or why not? (p. 454). Similarly, the following prompt from *Everyday Mathematics* also requires the student to use both *content* and *process* skills:

Feng said the name of the angle is SRT. Is he right? Explain (p. 8).

1) I know 4 $\frac{1}{1}$ is less than 2 $\frac{1}{2}$ because when comparing fractions that have a 1

in the numerator, you can look at the denominator. The larger the number in the denominator, the smaller the fraction.

2)
$$
\frac{1}{4}
$$
 is less than $\frac{1}{2}$

4) knowledge of the instructional strategies and representations for teaching particular topics (in mathematics) (p. 164).

Furthermore

opportunities for fostering meaningful connections between key concepts and

3) select particular students to present their mathematical work;

4) sequence the student responses that will be displayed in specific order; and

s and connect the responses to key

mathematical ideas.

Facilitation of writing prompts for purposes of discussion provides an opportunity for teachers and students to learn important mathematics content while enhancing the benefits of social interaction for learning.

Many mathematics educators and researchers view mathematics instruction as a social interaction process. For example, Steele (2009) notes the findings from Cobb, Yackel and Wood (1991) support children's opportunities to talk about their mathematical understanding. Students construct a more powerful way of thinking about mathematics through social interactions with a more knowledgeable person (p. 211). This knowledgeable person has the potential to be the teacher. In order for teachers to facilitate this type of environment where various responses are accepted for the same prompt, a thorough knowledge of the content should be acquired. This acquisition of knowledge in the form of professional development can also be conducted through the use of the Teacher Edition. For example, although textbooks are acknowledged as the dominant tool in the mathematics classroom for what is taught, they also have the value of providing professional development within their content (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr et al., 2008).

5. Teachers' Editions

I specifically analyzed the teacher editions of the two textbooks to provide insight as to the type of written support teachers receive regarding prompt instruction. I

examined each textbook for *support, sample, support with sample,* and *no support or sample* to gather data to the corresponding prompt coded in the student edition. If the prompt was coded in *support* then some form of directional support was provided to the teacher without a sample response. The category of *sample* identified prompts that only had support in the teacher edition in the form of a student sample response. The category of *support with sample* categorized prompts that had support in the teacher edition in the form of support with a sample student response. The final category of *no support or sample* signified that the teacher edition provided no support for the prompt.

Support and sample responses. Two of the most salient findings regarding the *teacher edition* are related to the *support* and *sample* categories. Both *enVision MATH* and *Everyday Mathematics* had the majority of prompts coded in the *sample* and *support with sample* categories

support at all. Furthermore, coding in this area implies that the teacher is left to his/her own discretion regarding instruction on the prompt. The novice teacher or one with low content knowledge in mathematics may find writing prompts coded in the area *no support or sample* a challenge to teach. However, after further examination, the ambiguity of the prompt affordance leaves the mathematics educator at a potential standstill regarding instruction. Although the teacher edition provided one sample response as the most common form of support the dilemma of how we treat these prompts in mathematics education remains a question.

This data is unsettling. The limited support for writing instruction in the teacher edition provides a key implication for textbook publishing companies. In an effo

4) Pick any three fractions in the box above and order from least to greatest. Next, pick one of the strategies listed in the strategy box to explain how you know your answer is correct.

These four prompts were developed in an instructional type of hierarchy. For example, the first problem relates to the patterning of fractions, the second relates to comparing and ordering fractions which is a little more complex than noticing a pattern. The third problem now asks the student to select a fraction larger than the one indicated. The request of justifying an answer using a guess and test will indicate that the student *should* select a few fractions to determine the correct solution, and the fourth problem allows the student to use fractions of choice and a strategy of choice. Furthermore, a student should not progress to the next problem in the sequence if there is an indication the problem cannot be solved. This type of formative assessment would provide a window into student thinking allowing for the teacher to assign tasks that are more complex based on the language or remediation before the next task in the textbooks can be attempted.

This type of hierarchy is based on Norman Webb (2002) three levels of cognitive complexity in mathematics tasks. For example, Level 1 mathematics items include the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. Level 2 mathematics items require students to make some decisions as to how to approach the problem or activity and Level 3 mathematics items require reasoning, planning, using evidence, and a higher level of thinking. The writing tasks mentioned are similar to these complexity levels whereby instruction would benefit by progressing through the levels in a type of

mathematical processes. Instruction regarding how to reflexively move from each element is encouraged as writing is a complex process. In support of a new paradigm for writing instruction in mathematics, Moje (2008) notes:

We need to consider the larger contexts in which strategies are drawn up and the practices that various strategies support. It may be most productive to build Disciplinary literacy instructional programs rather than merely encourage content teachers to employ literacy teaching practices and strategies (p. 96).

Additional research in these areas should be encouraged in order to fully implement writing in mathematics with success.

Types of curriculum: intended versus implemented. The *intended curriculum* is represented by goals and directives set forth in standards documents and policy, as well as their appearance in the teacher edition. The *implemented* curriculum is what actually is taught in the classroom (Schmidt et al., 2000; Valverde et al., 2002). Valverde et al. (2000) note:

The inclusion of a learning goal in the intended curriculum does not guarantee that it will be covered. Including an intention as a goal does not guarantee that the opportunity to attain that goal will actually be provided in the classroom but does greatly increase the probability that it will (p. 8).

Within this study, other influences could have a potential impact on what is implemented by the teacher and encountered by the student. However, these influences were not analyzed. Tarr, et al. (2008) note teacher knowledge and beliefs have the potential to impact the implemented curriculum. Although textbooks are acknowledged as the dominant tool in the mathematics classroom for what is taught, they have the value of

The first measure consisted of the percentage of agreement in choosing the same tasks as writing prompts. The second measure consisted of the percentage of agreement in choosing the same codes across framework dimensions.

The validity of the framework refers to how accurate the framework measures important features of writing prompts. A thorough review of extant literature regarding writing in mathematics coupled with reform recommendations provided direction regarding the development of the dimensions and categories across the framework. Although there were many forms of prompt affordances, only the prompts that provided a potential construction of more than a one-word answer were used for analysis in my framework.

Recommendations for future research

Aligned with reform efforts in mathematics instruction, new assessment tools based on two assessment consortia will require students to construct responses to literacy rich mathematical prompts as part of a national assessment in the near future. More specifically Shaughnessy (2011) noted:

The Partnership for the Assessment of Readiness for College and Career (PARC) and Smarter-Balanced Assessment Consortium (SBAC) have obtained federal grants to development assessment tools, both formative and summative, to assess students proficiency with the content and practices specified in the Common Core State Standards for mathematics (CCSSM) by the start of 2014 (NCTM Summing It Up, para.1).

Currently, states must decide which assessment consortia to adopt. Regardless of the states selection, both consortium will have students constructing a response to a

mathematical prompt as a measure of ability. Within this vein, mathematical literacy to including instruction in mathematical writing will be recommended. Results from my study coupled with the high stakes demand of writing in mathematics provide valuable information regarding five projected areas for future research.

The first area for future research would be to identify the different varieties of cognitive demands of writing prompts based on the language and vocabulary used in the prompts. Identifying if prompts are low level or high level in complexity according to

vels (Webb, 2002) ratings would inform the field of mathematics regarding the differentiation of writing tasks for instruction. Based on this information, writing task language could have the potential to be modified in order to increase the level of complexity or lower the level of complexity.

The second area for future research would be to include within an analytical type of framework coding for the graphics combined with the writing prompts. Identification of whether or not a graphic was used in the teacher edition could provide useful information regarding transference of information as another issue of complexity in composing a written construction.

The third area for future research would be to analyze student responses to mathematical writing prompts. Identification of the language within the prompts correlating to the language within the constructed response could have major instructional implication

from student explanation of answers, and teacher questioning could provide the field of

in mathematics.

Along the lines of teacher questioning, the final area for future research would be in the area of teacher instruction. Data regarding how teachers use the prompts and what teachers are really assigning in writing prompts would be worth knowing. For example, using mathematical writing prompts at the beginning, middle or end of a lesson would also inform teachers regarding the most appropriate application of mathematical writing

As students are asked to communicate about mathematics they are studying to justify their reasoning to a classmate or to formulate a question about something that is puzzling they gain insights into their thinking. In order to communicate their thinking to others, students naturally reflect on their learning and organize and consolidate their thinking about mathematics. (p. 63).

Similarly, the *Curriculum Focal Points* (NCTM, 2006) also support the use of writing in mathematics through the implementation of reasoning, justification and communicating. Additionally, the NRC developed interrelated strands for mathematical proficiency integrating the use of writing. Further recommendations through the CCSS also support the use of writing within the *Standards for Mathematical Practice* (CCSS, 2010).

This study was developed to inform the field of mathematics how textbooks support these reform recommendations of writing in mathematics through an investigation of writing prompts. Additionally, textbooks are known to have an influence on classroom instruction since they are used often as instructional tools (Ball $\&$ Cohen, 1996). An investigation of the prompt affordances through an analysis of the vocabulary and language used in the mathematical prompt stems provided salient discussion regarding the complexity of instruction and composition in this area and the implications

 Research regarding best practices in vocabulary instruction relating to literacy should help inform the field of mathematics regarding the importance of integrating such strategies. Additionally, the a priori word lists should be updated and revised to include the different derivatives and word associations of vocabulary needed in order to communicate mathematically. These derivatives and associations of words have the potential to create abstract meanings.

The lack of support found in the teacher edition for these types of prompts is a clear indication that the area of teacher support for writing in mathematics needs to be reconsidered in the teacher editions. The first reason for this implication is that the complexity of the language of the mathematical prompts stems, coupled with the vocabulary, indicates these prompts are ambiguous in nature. The ambiguity of these prompts allows for various processes to be used therefore providing many opportunities for variety of responses.

Differences in the textbooks were also discussed. In light of the finding that the *enVision MATH* had more writing prompts coded, there were more overall exercises for students to encounter. The large amount of exercises in this textbook could affect what teachers choose to assign and instruct. If teachers are unfamiliar with the content and find the support lacking in the teacher edition regarding prompt directions, the writing prompts *may* be skipped. The omission of tasks, due to teacher selection, could affect student potential opportunity to learn.

Because the mathematics textbook is researched as the dominant tool in classroom instruction (Hagarty & Pepin, 2002; Johansson, 2005; Malzahn, 2002; Schmidt, 2004; Tarr, et al., 2008), it was encouraging to find that textbook developers are

adhering to reform recommendations of writing in mathematics. Although the textbooks explored are different in their philosophies, there were a few recommendations for both textbooks in order to improve student textbooks and teacher editions. These recommendations welcome the collaboration of literacy and mathematics researchers and experts in order to develop the instructional tools needed for successful implementation of writing in mathematics. Discussions centered upon the following five ideas would be constructive regarding the development of textbooks and instructional materials: 1) vocabulary used in the prompts and the types of vocabulary needed to facilitate potential response, 2) the multiple strategies and processes that could potentially be used by students in order to construct a response, 3) teacher development resources coupled with the teacher edition regarding the variety of prospective answers, 4) teacher development resources regarding prompt instruction using a triage approach, 5) development of a balanced number of writing prompts in all content areas. This collaborative union would benefit the fields of both literacy and mathematics.

Before we can begin to implement the process of writing in the mathematics classroom, a love for the discovery of mathematical knowledge through the mere act of communication should be embraced in all facets within the teaching and learning of mathematics.

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Appendices

Appendix B: List of Terms for Identification of Prompts

Appendix C: Pilot Framework (revisions in bold)

Appendix E: Vocabulary Classification Scheme

Appendix F: Domain Specific Academic Vocabulary

2-dimensional shape 2-dimensional shape combination 2-dimensional shape decomposition 2-dimensional shape slide 2-dimensional shape turn 2-dimensional space 3

inch increasing pattern independent events independent trials

open sentence order of operations ordered pairs ordinal number orientation outcome outliers outside overestimation parallel box plot parallel lines parallelogram parallelogram formula parameter parameter estimate parametric equation part of whole path pattern pattern addition pattern division pattern extension pattern multiplication pattern recognition pattern subtraction percent percents above 100 percents below 1 perimeter perimeter formula periodic function permutation perpendicular bisector perpendicular lines perspective phase shift pi pictorial representation pie chart place holder planar cross section plane plane figure point of tangency polar coordinates

polygon polynomial polynomial addition polynomial division polynomial function polynomial multiplication polynomial solution by bisection polynomial solution by sign change polynomial solution successive approximation polynomial subtraction population positive number postulate pound powers precision of estimation precision of measurement prediction prime factor prime factorization prime number prism probability probability distribution problem formulation problem space problem types process of elimination product projection proof proof paragraph proportion proportional gain protractor pyramid pythagorean theorem quadratic equation quadrilateral quartile deviation quotient

representativeness of sample restate a problem reversing order of operations rhombus richter scale right right angle right triangle geometry roman numeral root roots & real numbers roots to determine cost roots to determine profit roots to determine revenue rotation rotation in plane rotation symmetry rounding ruler same size units sample sample selection techniques sample space sample statistic sampling distribution sampling error scalar scale scale drawing scale map scale transformation scatter plot scientific notation second (time) sequence series series circuit set shape combination shape division shape pattern shape similarity

shape symmetry shape transformation shrinking pattern shrinking transformation sigma notation significant digits similar figures similar proportions similarity similarity vs. congruence simplification sine sinusoidal function size slide transformation slope slope intercept formula smallest set of rules solid figure solution algorithm solution probabilities sound attern speed sphere spreadsheet spurious correlation square square number square root square units standard deviation standard measure of weight standard measures of time standard vs. non standard units statistic statistical experiment statistical regression stem & leaf plot step function straight edge & compass

triangle formula triangle sides trigonometric ratio trigonometric relation truncation truth table proof two way tables u.s. customary system under underestimation unit analysis unit conversation unit differences unit size univariate data univariate distribution unknown unlike denominators upper/lower bounds valid argument

validity variability variable variable change variance vector vector addition vector division vector multiplication vector subtraction velocity venn diagram verbal representation of a problem verification vertex vertex edge graph vertical axis volume volume formula

volume measurement volume of cylinder volume of irregular shapes volume of prism volume of pyramid volume of rectangular solids week whole number width work backward written representation year zero

Appendix G: General Academic Vocabulary Word List

abandon abstract academy access accommodate accompany accumulate accurate achieve acknowledge acquire adapt adequate adjacent adjust administrate adult advocate affect aggregate aid albeit allocate alter

coopera223

imply impose incentive incidence incline income incorporate index indicate individual induce inevitable infer infrastructure inherent inhibit initial initiate injure innovate input insert insight inspect instance institute instruct integral integrate integrity intelligence intense interact intermediate internal interpret interval intervene intrinsic invest investigate invoke involve isolate issue item

job journal justify label labor layer lecture legal legislate levy liberal license likewise link locate logic maintain major manipulate manua l margin mature maximize mechanism media mediate medical medium mental method migrate military minimal minimize minimum ministry minor mode modify monitor motive mutual negate network neutral nevertheless nonetheless norm norma l notion notwithstanding nuclear objective obtain obvious occupy occur odd offset ongoing option orient outcome output overall overlap overseas panel paradigm paragraph parallel parameter participate partner passive perceive percent period persist perspective phase phenomenon philosophy physical plus policy portion pose positive potential practitioner precede

precise predict predominant preliminary presume previous primary prime principal principle prior priority proceed process professional prohibit project promote proportion prospect

ultimate undergo underlie undertake uniform unify unique utilize valid vary

appeal to emotion appeal to logic appendix

acronym action segment action verb action word active listener actor adjective adjective clause adjective phrase adverb adverb clause adverb phrase advertisement advertising code advertising copy aesthetic purpose aesthetic quality affix allegory alliteration allusion almanac alphabet ambience ambiguity american literature american psychological association analogy ancient literature anecdotal scripting anecdote anglo-saxon affix anglo-saxon root animation annotated bibliography antonym apology apostrophe appeal to authority

argumentation articulation artifact asking permission assonance atlas attack ad hominem audience

common noun comparative adjective compare & contrast compile complete sentence complex sentence composition composition structure compound adjective compound noun compound personal pronoun compound sentence compound verb compound word compound-complex sentence comprehension computer generated image concept conceptual map concluding statement conclusion conjunction conjunctive adverb connotative meaning consonance consonant blend consonant substitution construct meaning consumer document content-area vocabulary context context clue contract contraction contrast contrasting expressions controlling idea convention conversation coordinating conjunction copyright law correlative conjunction counter argument couplet

cover credibility credit criteria critical standard criticism cross-reference cue cultural agency cultural expression cultural influence cultural nuance cultural theme current affairs cursive custom cutline dash date debate declarative sentence decode deconstruct definition delivery demonstrative pronoun denotative meaning derivation description descriptive language detail diagram dialect dialogue diary dictation dictionary dictionary digressive time direct address direct quote directionality directions director discussion discussion leader

divided quotation document documentary double negative draft drama drama-documentary dramatic dialogue dramatic mood change drawing edit editorial elaboration electronic media e-mail emotional appeal emphasis encyclopedia ending ending consonant enunciation epic episode essay ethics etiquette etymology everyday language exaggerated claim example excerpt exclamation mark exclamatory sentence explanation explicit/implicit exposition expression expressive writing extend invitation extended quotation external/internal conflict extraneous information eye contact fable facial expression facilitator

specialized language speech action speech pattern speed reading speed writing spelling spelling pattern spoken text standard english status indicator stay on topic stereotype story element story map story structure stream of consciousness stress structural analysis style sheet format stylistic feature sub vocalize subject subject pronoun subjective view subject-verb agreement subliminal message subordinate character subordinating connection subplot suffix summarize summary summary sentence superlative adjective

Appendix I: Meta-language Academic terms for Book Parts Word List

author index bibliography boldface type caption chapter chart column conclusion diagram excerpt figure font size font/print glossary graph (line/bar) graph (pie) handbook illustration/picture indentation index introduction italicized type map page paragraph passage preface quotation section selection subtitle/subheading table table of contents title heading title page transition

- (open parenthesis
- [open bracket
- @ at
- : is to therefore
- r set of real numbers

union with or union contained in or a subset of element of equivalent

- minus or negative
- \div is divided by
	- is not equal to
- > is greater than
- / is divided by
- \angle angle
- \perp is perpendicular to is greater than or equal to closed parenthesis
-] closed bracket
- ø null set, empty set or zero
- :: as

is approximately

- n set of natural numbers intersects or intersection
- σ not a subset of is not an element of is parallel to
4) Read the prompt to determine if the language in the prompt provides for an additional strand to be selected. See Table 18) If a word is not found in the Academic Vocabulary word list the rater may code the word in the Special Words section of the Framework with the appropriate classification of the Academic Vocabulary next to the word.

Meta Language (ML)

Identify words that are specific to ML by recognizing words in the prompt that are used to describe the language of literacy and literacy instruction and words used to describe processes, structures, or concepts commonly included in content area texts.

- 1) Review the terms in Appendix H-I to assist in the identification of ML.
- 2) Conduct a word search using the Ctrl Find Key in the Excel Spreadsheet of Academic Vocabulary Word Lists. Words are color coded according to the categories in the Academic Vocabulary section of the Framework.
- 3) Continue with the Ctrl Find key until you have exhausted the search and returned back to the initial position.
- 4) Record findings in the appropriate Academic Vocabulary sections in the Framework.
- 5) If a word is found in two or more Academic Vocabulary sections the word is coded appropriately in each section and underlined.
- 6) Identified words may be derivatives of the Academic Vocabulary found in the Word Lists. The derivative is noted next to the word coded in parentheses.
- 7) If a word is not found in the Academic Vocabulary word list the rater may code the word in the Special Words section of the Framework with the appropriate classification of the Academic Vocabulary next to the word.

Symbols

- 1) Words in the prompt are NOT mathematics symbols.
- 2) Punctuation marks in the prompt are NOT mathematics symbols (i.e., commas including seriations (lists), hyphens used between words, periods, and question marks).
- 3) All numerals that represent numbers will be coded as symbols.
- 4) Any symbol that is NOT a word or part of the punctuation in the prompt will be coded as a symbol.
- 5) If a symbol is combined with another symbol the symbol will be coded as *one.* The parts that make the symbol, if those parts are in the symbols list, will also be counted independently.

Words Not On List

This section is not co-rated.

Total

Words and Symbols Coded

- 1) Count the number of words and symbols coded in each of the Academic Vocabulary sections.
- 2) Numerals are coded as symbols and counted as one number.
- 3) Commas, periods, colons, dollar signs, fraction symbols, within numbers are counted as one symbol *and* as individual symbols. For example (2,000,567 is counted as 3, one time for the whole number and two times for each comma).
- 4) Phrases are counted as individual words.
- 5) Underlined words are counted one time.
- 6) Special Words category is NOT counted.

Words and Symbols

- 1) Count the total number of all words and all symbols in the prompt.
- 2) Commas, periods, colons, dollar signs, fraction symbols, within numbers are counted as one symbol *and* as individual symbols. For example (2,000,567 is counted as 3, one time for the whole number and two times for each comma).
- 3) Phrases are counted as individual words

Type of Prompt

- 1) Because all the prompts coded are generic, the prompt will only be coded in this section if it is NOT coded in the other categories.
- 2) Affective prompts are coded in this section if the prompt involves the reader to write an opinion, feeling, or belief regarding the topic.
- 3) Narrative is coded in this section if the prompt provides the writer with information to write about math content in a fictional or narrative sense using real world or imaginary indicators.

Teacher Edition

Find the section of the Teacher Edition for the prompt coded. Read the section carefully to indicate the following codes listed below.

Support provided only (Su)

1) A prompt is coded in this section if the Teacher Edition only has teaching support for the prompt coded. Support includes any indicator of instructional notes for the prompt. Any information given to the teacher for the prompt other than a student sample is coded in this section.

Sample provided only (Sa)

1) A prompt is coded in this section if the Teacher Edition only has a sample of how the prompt should be answered for the prompt coded. No other directions or guidance is given for the prompt.

Support with Sample provided (SS)

1) A prompt is coded in this section if the Teacher Edition has both teaching support *and* a sample of how the prompt should be answered for the prompt coded.

No Support or Sample provided (N)

1) A prompt is coded in this section if the Teacher Edition has NO teaching support or sample answer provided.

Student Edition (SE)

This section is not co-rated.

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Review:

Fractions,

Decimals, and

Percents

Permissions and IRB Application

Permissions

May 18, 2011

Good afternoon Christine,

Thank you for your email!