



Using Research to Improve Instruction

Karen Karp
Volume Editor
University of Louisville
Louisville, Kentucky

Amy Roth McDuffie
Series Editor
Washington State University Tri-Cities
Richland, Washington



Copyright © 2014 by
The National Council of Teachers of Mathematics, Inc.
1906 Association Drive, Reston, VA 20191-1502
(703) 620-9840; (800) 235-7566; www.nctm.org
All rights reserved

ISSN 2332-6336
ISBN 978-0-87353-761-2

CONTENTS

Preface..... v

PART I Change..... 1

1.1.1.1..... 1

1.1.1.2..... 1

1.1.1.3..... 1

1.1.1.4..... 1

1.1.1.5..... 1

1.1.1.6..... 1

1.1.1.7..... 1

1.1.1.8..... 1

1.1.1.9..... 1

1.1.1.10..... 1

1.1.1.11..... 1

1.1.1.12..... 1

1.1.1.13..... 1

1.1.1.14..... 1

1.1.1.15..... 1

1.1.1.16..... 1

1.1.1.17..... 1

1.1.1.18..... 1

1.1.1.19..... 1

1.1.1.20..... 1

1.1.1.21..... 1

1.1.1.22..... 1

1.1.1.23..... 1

1.1.1.24..... 1

1.1.1.25..... 1

1.1.1.26..... 1

1.1.1.27..... 1

1.1.1.28..... 1

1.1.1.29..... 1

1.1.1.30..... 1

1.1.1.31..... 1

1.1.1.32..... 1

1.1.1.33..... 1

1.1.1.34..... 1

1.1.1.35..... 1

1.1.1.36..... 1

1.1.1.37..... 1

1.1.1.38..... 1

1.1.1.39..... 1

1.1.1.40..... 1

PART II Problem Solving..... 42

2.1.1.1..... 42

2.1.1.2..... 42

2.1.1.3..... 42

2.1.1.4..... 42

2.1.1.5..... 42

2.1.1.6..... 42

2.1.1.7..... 42

2.1.1.8..... 42

2.1.1.9..... 42

2.1.1.10..... 42

2.1.1.11..... 42

2.1.1.12..... 42

2.1.1.13..... 42

2.1.1.14..... 42

2.1.1.15..... 42

2.1.1.16..... 42

2.1.1.17..... 42

2.1.1.18..... 42

2.1.1.19..... 42

2.1.1.20..... 42

PART III Reasoning, Explaining, and Discourse..... 100

3.1.1.1..... 100

3.1.1.2..... 100

3.1.1.3..... 100

3.1.1.4..... 100

3.1.1.5..... 100

3.1.1.6..... 100



12. Situating Expansions of Students' Explanations in Discourse Contexts119

Surface

13. Multimodal Communication: Promoting and Revealing Students' Mathematical
Thinking 129

PART IV Seeing Structure and Generalizing.....

PART V Assessment for Teaching and Learning

[Redacted text]

[Redacted text]

[Redacted text]

■ In Appreciation

First and foremost, the Editorial Panel members want to thank the [Redacted]

selecting chapters for, editing, and printing each yearbook. Moreover, the time from when

that the panel decided were at the forefront of mathematics education: Change; Problem Solving;

[Redacted text]

[Redacted text]

[Redacted text]

[Redacted text]

[Redacted text]

[Redacted text]

[Redacted text]

■ Creating the Inaugural Volume

[Redacted text]

[Redacted text]

[Redacted text]

[Redacted text]

[Redacted text]

[Redacted text]

Promoting Mathematical Reasoning through Critiquing Student Work

Catherine Bénéteau, *University of South Florida, Tampa*
Sarah K. Bleiler, *Middle Tennessee State University, Murfreesboro*

Mathematical reasoning, eventually resulting in formal proof, is an essential component of mathematical learning and can range “from informal explanation and justification to formal deduction, as well as inductive observations. Reasoning often begins with explorations, conjectures at various levels, false starts, and partial explanations before a result is reached”

(National Council of Teachers of Mathematics [NCTM], 2000, p. 66)

■ Context

A group of mathematicians and mathematics educators collaborated to design and teach a geometry course and then a subsequent semester with many of the same PSTs. Within geomet

PSTs would learn mathematics through an inquiry approach.

Because one goal was to improve PSTs' ability to understand and write proofs, our critique tasks addressed two areas where PSTs demonstrated misconceptions. The first area focused on the

■ Critiquing Sample Student Work

the central angle subtended by the two arcs is the same angle. This response could have been even stronger had the student clearly indicated that the two arcs were not congruent, so completeness is not an absolute standard but falls along a continuum.

Thus, this task forced the PSTs to examine not only mathematical inaccuracies (criteria 1) but

Sample Tasks Involving the Identification of Hypotheses and Conclusions

As we began instruction on proof, we realized that PSTs struggled with identifying the hypotheses and conclusions of statements. Sometimes, as instructors, we spend considerable time trying to

communicate sophisticated ideas relative to proof development, such as ideas related to appropriate modes of argumentation (e.g., direct and indirect reasoning) or modes of argument representation (e.g., symbolic or narrative), when the basic concepts of the initial assumptions and the conclusion

are not clear to our students. As a preliminary task relative to this issue, PSTs examined sample responses in which the theorem “Bisectors of supplementary angles are perpendicular” had been written in if-then form. Indeed, the statement of the theorem appears to cause PSTs to reflect

A B (1) Since $\angle 1$ and $\angle 2$ are alternate interior angles
 C $AB \parallel DC$.
 D
 (2) Since the diagonals bisect each other, $AO \cong OC$ and $DO \cong OB$.
 $\angle AOD$ and $\angle BOC$ are vertical angles,

the room, without names. In groups, the PSTs circulated to read the work of C...

Implications for Practice: Critiquing Sample Work to Focus on Metaknowledge about Proof

Advancing Critiques of Sample Work

We used varied implementation approaches to engage PSTs with critiquing sample student work.

(4) ... PSTs discuss sample responses in groups and

1. _____



■ PART IV

Seeing Structure and Generalizing